# Efficient Wireless Power Transfer – Resonance Does Not Imply High Efficiency

Kazuya Yamaguchi, *University of Miyazaki* Ichijo Hodaka, *University of Miyazaki*

*Abstract*—This paper naively inquires when power transmission is optimized by tuning possible parameters of a wireless power transfer system. It is well known that tuning the frequency of the AC supply input implies maximizing the magnitude of transmitted power into a receiver side. This type of tuned wireless power transfer system, however, may cause a low efficiency of power transfer. The paper illustrates that such an undesired situation exists, and suggests that tuning parameters in the both views of large magnitude of transmitted power and high efficient transmission will be desirable for better wireless power transfer systems.

*Keywords*—wireless power transfer, highly efficient power transmission, resonance

## I. INTRODUCTION

MOST of devices that are driven by electric power are usually fed via electric wires connected to AC power supply. On the other hand, some of electric devices are required or designed to be fed without electric wires — in the way of called WPT (wireless power transfer). Since the amount of power provided by WPT is usually restricted compared with wired power transfer, WPT has not been used for many applications.

In 2007, a successful experiment that could wirelessly transfer practical amount of power away from sixty centimeters was reported[1]. The heart of the experiment was to generate resonant phenomena of electromagnetic coupling between transmitting and receiving coils. After the pioneering study[1], many researchers have tried to develop theory and experiments for WPT. In [2] a structure of circuit was devised in order to raise the factor Q of circuits, since better WPT needs higher values of the factor Q. In [3] devised antennas are proposed to be used for a directed energy radiation in order to implement efficient WPT. In [4] the effect of radiation energy to human body was discussed.

It is generally understood that resonant phenomena will be caused if one tunes the frequency of AC power supply to the natural frequency of a circuit. The natural frequency is determined by the values of circuit elements; especially, the mutual inductances which relate transmitting and receiving sides of the circuit play an important role. These values are determined by the radii, winding numbers, and relative position of two coils[5]. It is important to know the relation between the relative position of coils and mutual inductances because the position of the coils could be changed in various reasons.

For extending the usage of WPT, the most important specification for WPT is power and efficiency of transmission. In many literatures, causing resonant phenomena should lead to high power and efficient WPT. However, little is known about how resonance brings the best performance of WPT based on a concrete and mathematical aspects.

In this paper, we propose a series of procedures to analyze relations between the resonant frequency, obtained average power at the receiving side, and the ratio of the average power at the receiving side to the average power at the transmitting side that is called an efficiency of WPT. Our procedures are based on the differential equations in the form which commonly used in control theory. This gives us a benefit that we can manipulate different transfer functions and write the relations under investigation in a clear and unified manner. Consequently, we will assert that resonance does not imply the optimization of efficiency of WPT in general, by observing derived equations.

To illustrate this situation that should be carefully treated, we pick one of most common circuits for WPT, and put practical values into the circuit elements, and then we demonstrate the situation numerically. In fact, although we have the case when resonance is equivalent to maximization of efficiency of WPT, we have another case when resonance leads to loss of efficiency of WPT. Thus we propose to use the way of modelling by mathematical equations and to describe the targets into equations for WPT systems.

## II. ANALYSIS OF WIRELESS POWER TRANSFER CIRCUIT

### A. Self inductance and mutual inductance

We consider the situation that current $i$ flows in a coil with a radius $r_1$ and a winding number $n_1$, called coil 1. This generates magnetic flux in the whole space by the Biot-Savart law. Here we consider a magnetic flux density $B$ which is away from the current $i$ by $z$ and which is along the central axis of the coil 1[6]. That is,

$$B = \frac{\mu i}{4\pi}\frac{2n_1\pi r_1^2}{(z^2+r_1^2)^{3/2}}. \tag{1}$$

Where $\mu$ is the permeability.

Now we consider another coil 2 with a radius $r_2$ and a winding number $n_2$. From equation (1), self inductances $L_1$ of the coil 1 and $L_2$ of the coil 2, and mutual inductances $M_1$ and $M_2$ between the coils 1 and 2 can be written as below.

$$\begin{aligned} &L_1 = \frac{\mu\pi n_1^2 r_1}{2},\ L_2 = \frac{\mu\pi n_2^2 r_2}{2} \\ &M_1 = \frac{\mu\pi n_1 n_2 r_1^2 r_2^2}{2(z^2+r_1^2)^{3/2}},\ M_2 = \frac{\mu\pi n_1 n_2 r_1^2 r_2^2}{2(z^2+r_2^2)^{3/2}}. \end{aligned} \tag{2}$$

From the above equations, note that these mutual inductances are inversely proportional to cube of the distance $z$ between two coils[7].

### B. Wireless power transfer circuit and its mathematical model

We study a typical circuit for WPT depicted below[8]. Despite of the placement of two coils in the figure, we assume they have a common central axis as explained in the previous section.

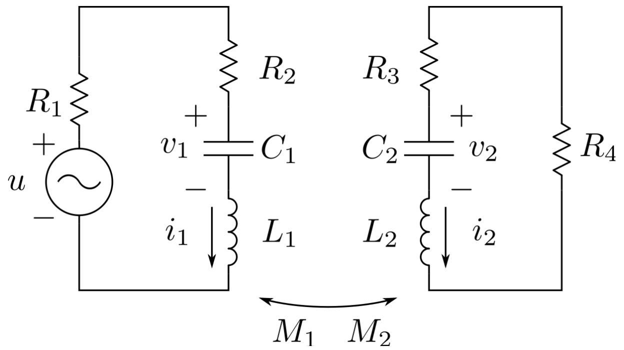

Figure 1 A Wirelss Power Transfer Circuit

The resistor $R_1$ is supposed to represent an internal impedance of the power supply, and $R_4$ the load. $R_2, R_3, C_1, C_2$ are parasitic factors of transmitting and receiving coils. This circuit is mathematically modelled as the following state equation. This type of expression is widely used in control theory. In particular one can write the model of WPT circuits in a compact form, and obtain a clear perspective to analysis of stability and responses with a sinusoidal input.

$$
\begin{aligned}
&\dot{x} = Ax + Bu \\
&x = \begin{bmatrix} v_1 \\ v_2 \\ i_1 \\ i_2 \end{bmatrix} \\
&A = \frac{1}{\Delta}\begin{bmatrix} 0 & 0 & \frac{\Delta}{C_1} & 0 \\ 0 & 0 & 0 & \frac{\Delta}{C_2} \\ -L_2 & M_2 & -(R_1+R_2)L_2 & (R_3+R_4)M_2 \\ M_1 & -L_1 & (R_1+R_2)M_1 & -(R_3+R_4)L_1 \end{bmatrix} \\
&B = \frac{1}{\Delta}\begin{bmatrix} 0 \\ 0 \\ L_2 \\ -M_1 \end{bmatrix} \\
&\Delta = L_1 L_2 - M_1 M_2.
\end{aligned} \tag{3}
$$

### C. Formulation of average power and efficiency

The purpose of this paper is to investigate the relation between the frequency of AC supply voltage, the average power delivered to the receiving side, and the efficiency of average power transmission. The efficiency is defined as the ratio of the average power obtained at the receiving side against the average power supplied at AC power supply. Thus we formulate equations of the average powers at transmitting and receiving sides, and of the efficiency in the following.

Since the matrix $A$ in the equation $(3)$ is stable, i.e., all eigenvalues of the matrix A have negative real parts, the solution to the equation $(3)$ will be stationary after time have passed adequately. The stationary solution with a sinusoidal input $u = \sin\omega t$, will have the same frequency of the input.

In general, the average powers $P_1$ and $P_4$ which are at the AC supply and the load in the receiving side respectively, and then the efficiency $\eta$ depends on the frequency of AC supply. These stationary values can be all expressed in terms of transfer functions.

$$
\begin{aligned}
P_1 &= \frac{1}{2}\left|\mathrm{Re}[G_1(j\omega)] - R_1|G_1(j\omega)|^2\right| \\
P_4 &= \frac{1}{2}R_4\left|G_2(j\omega)\right|^2 \\
\eta &= \frac{P_4}{P_1}.
\end{aligned} \tag{4}
$$

Here $G_1(s)$ and $G_2(s)$ are the transfer functions from the input $u$ to $i_1$ and $i_2$, respectively. With the equation (3), these transfer functions are written in $G_1(s) = H_1(sI - A)^{-1}B, G_2(s) = H_2(sI - A)^{-1}B$, where $H_1 = [0,0,1,0]$ and $H_2 = [0,0,0,1]$.

### D. Resonance and average power

In view of the equation $(4)$, we see that if one puts an AC input voltage with the frequency which gives the maximal gain of the transfer function $G_2(s)$ (the gain is called $H_\infty$-norm), one will have the maximal average power at the receiving side. To illustrate the situation, we set values of circuit elements as below. These values are given by consulting a practical situation of WPT[2].

TABLE I
PARAMETERS

| elements | values | elements | values |
|---|---|---|---|
| $R_1$ | $50\Omega$ | $L_2$ | $10\mu$H |
| $R_2$ | $0.1\Omega$ | $M_1$ | $0.5\mu$H |
| $R_3$ | $0.1\Omega$ | $M_2$ | $0.5\mu$H |
| $R_4$ | $50\Omega$ | $C_1$ | 1nF |
| $L_1$ | $10\mu$H | $C_2$ | 1nF |

The bode diagram of $G(s)$ is shown as below.

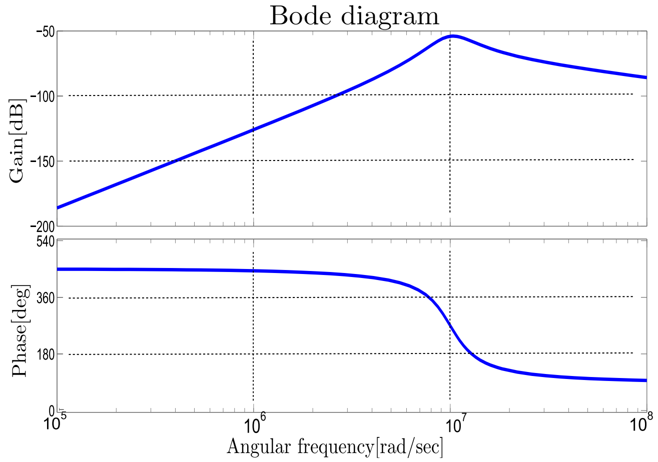

Figure 2 Bode Diagram

In this case the resonance will occur at the unique frequency $\omega = 1.00 \times 10^7 \mathrm{rad/sec}$ and we have the maximal average power at the receiving side if we use a sinusoidal wave with the resonant frequency.

### *E. Resonance and efficiency*

Based on the equation $(4)$, we can observe that the relation between the input frequency and the efficiency of WPT is not straightforward. Many other papers state that using an AC input voltage with a resonant frequency is optimal or better in the design of WPT. In the previous section, we have clarified that using the input with resonance maximizes the average power at the receiving side. However, the efficiency of power transmission can be maximized either when one use resonance or when one use nonresonance. This difficult situation is illustrated by numerical examples in the following.

First, power $P_4$ and efficiency $\eta$ with the condition of TABLE I are shown as below.

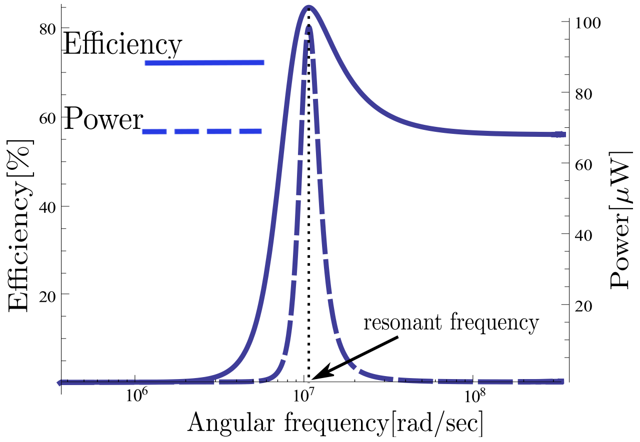

Figure 3 Power and Efficiency for TABLE I

Then, $\eta$ is maximized when

$$\omega = 1.07 \times 10^7 \mathrm{rad/sec} \tag{5}$$

as previously known in [9].

Another example is shown as below. The values of elements are set as TABLE II. The only difference between TABLE I and TABLE II is the value of $C_1$.

TABLE II
PARAMETERS

| elements | values | elements | values |
|---|---|---|---|
| $R_1$ | $50\Omega$ | $L_2$ | $10\mu$H |
| $R_2$ | $0.1\Omega$ | $M_1$ | $0.5\mu$H |
| $R_3$ | $0.1\Omega$ | $M_2$ | $0.5\mu$H |
| $R_4$ | $50\Omega$ | $C_1$ | 0.1nF |
| $L_1$ | $10\mu$H | $C_2$ | 1nF |

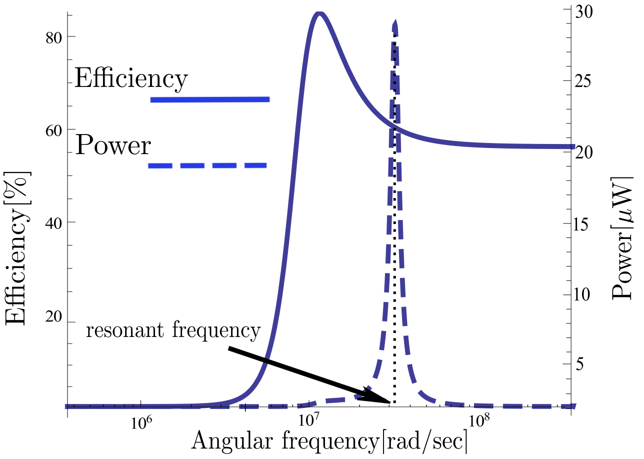

Figure 4 Power and Efficiency for TABLE II

By comparing Figure 3 and Figure 4, we see that the frequencies which respectively maximize power and efficiency are different.

## III. CONCLUSION

In this paper, we have pointed out that using a resonant frequency of AC voltage input does not lead to maximize the efficiency of average power transmission, although resonance is equivalent to maximization of output average power, in general WPT systems. In fact, we have illustrated a situation that non-resonance maximizes the efficiency of power transmission by numerical examples. This suggests that we should take both of output power and efficiency into account and then decide a balanced working frequency, in the case that resonance is not equivalent to the maximal efficiency. For example, the maximum power is desirable if an AC power supply can serve enough power, but the maximum efficiency is best if the supply can serve less power.

Even one of the simplest WPT circuits treated in this paper may have disagreement between resonance and efficiency. Therefore, one will face on further difficulty to design better WPT when one tries more complex circuit with more elements in order to meet an increasing design specification.

## REFERENCES

[1] Andr Kurs, Aristeidis Karalis, Robert Moffatt, J. D. Joannopoulos, Peter Fisher, Marin Soljacic´ , ”Wireless Power Transfer via Strongly Coupled Magnetic Resonances”, Science 317, PP.83-86, 2007.

[2] Huy Hoang, Franklin Bien, ”Maximizing Efficiency of Electromagnetic Resonance Wireless Power Transmission Systems with Adaptive Circuits”, Wireless Power Transfer - Principles and Engineering Explorations, PP.207-226, 2012.

[3] Ick-Jae Yoon, Hao Ling, ”Realizing Efficient Wireless Power Transfer in the Near-Field Region Using Electrically Small Antennas”, Wireless Power Transfer - Principles and Engineering Explorations, pp.151-172, 2012.

[4] Ji-Yeon Mun, Min-Gyeong Seo, Woo-Geun Kang, Hae-Young Jun, Yong-Ho Park, Jeong-Ki Pack, ”Study on the Human Effect of a Wireless Power Transfer Device at Low Frequency”, PIERS Proceedings, pp.322-324, 2012.

[5] Hiroshi Hirayama, ”Equivalent Circuit and Calculation of Its Parameters of Magnetic-Coupled-Resonant Wireless Power Transfer”, Wireless Power Transfer - Principles and Engineering Explorations, pp.117-132, 2012.

[6] Youngjin Park, Jinwook Kim, Kwan-Ho Kim, ”Magnetically Coupled Resonance Wireless Power Transfer (MR-WPT) with Multiple Self-Resonators”, Wireless Power Transfer - Principles and Engineering Explorations, pp.51-64, 2012.

[7] Henri Bondar, Shailendra Oree, Zafrullah Jagoo, Keiichi Ichikawa, ”Estimate of the maximum range achievable by non-radiating wireless power transfer or near-eld communication systems”, Journal of Electrostatics, Volume71, Issue4, pp.648-655, 2013.

[8] Marco Dionigi, Alessandra Costanzo, Mauro Mongiardo, ”Network Methods for Analysis and Design of Resonant Wireless Power Transfer Systems”, Wireless Power Transfer - Principles and Engineering Explorations, pp.65-94, 2012.

[9] Takashi Komaru, Masayoshi Koizumi, Kimiya Komurasaki, Takayuki Shibata, Kazuhiko Kano, ”Compact and Tunable Transmitter and Receiver for Magnetic Resonance Power Transmission to Mobile Objects”, Wireless Power Transfer - Principles and Engineering Explorations, pp.133-150, 2012.