New Tuning Method of the Wavelet Function for Inertial Sensor Signals Denoising

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Abstract — The current research is aimed at implementing and validating software procedures by proposing new algorithms that receive data from inertial navigation systems/sensors (data acquisition equipment) and provide accurate navigation information. In the first phase, our concern was to implement a real-time evaluation criterion with the intention of achieving real-time data from an accelerometer. It is well known that most errors in the detection of position, velocity and attitude in inertial navigation occur due to difficult numerical integration of noise. The main goal of this study was to propose a signal processing algorithm, based on the Wavelet filter, used along with a criterion for evaluating and updating the Wavelet's optimal level of decomposition, for reducing the noise component. We performed numerical simulations using signals received from an accelerometer and analyzed the numerical results to see whether the improved Wavelet (proposed method) can be used to achieve more precise information on a vehicle.

Keywords **—** signal processing; wavelet transform, partial directed coherence method.

I. Introduction

The aim of the our scientific research is to develop advanced algorithms able to determine the optimal level of decomposition for the Wavelet method and to implement these algorithms in a miniaturized inertial measurement units in order to obtain accurate data regarding the vehicle displacement.

Apart from the indisputable benefits of size reduction, reliability, manufacturing costs and power consumption, the miniaturization of sensors and by default of inertial measurement systems (*INS*) caused a number of problems related to their performance degradation. As a result of miniaturization, stochastic (noise of the system) and deterministic errors occurred [1-4].

The inertial sensors noise, major source of errors for inertial navigation systems, is characterized by a constant power throughout the frequency spectrum, that reflects the dynamics of mobile systems - which are intended to be monitored (generally in the range 0-100 Hz). Therefore, this type of noise filtering in the 0-100 Hz band is not recommended.

This noise component that overlaps over the output of the sensors, cannot be totally eliminated but it can be influenced by stochastic processes [5].

The development and also the optimization of advanced algorithms for improving the performances of miniaturized inertial sensor and inertial measurement units are extremely important topics in the field of aerospace navigation systems.

The practical challenge of the study under discussion was to develop and validate a complex algorithm that would process the signals received from accelerometers, and later from *INS*, remove the noise and offer precise information regarding the vehicle displacement.

Therefore, an improved wavelet filter was proposed, used to remove the noise detected during measurements, in order to obtain a better accuracy of the measurements. The optimal order of the wavelet filter (the optimal decomposition level) was calculated using a correlation analysis function applied to the signals achieved from the accelerometers and the real speed signals applied to the accelerometers (considered as reference signals).

II. Proposed Method

The Wavelet transform is a very powerful tool for the signal feature extraction and noise reduction and offers effective localization in time and frequency domains.

In order to analyze signals, the continuous Wavelet transform (*CWT*) can be considered as a tree decomposition of the signal (the Wavelet decomposition tree), a combination of a set of basic functions, obtained by means of dilation and translation of a single prototype Wavelet function $\psi(t)$ called the mother Wavelet, as illustrated in Fig. 1 [6].

In practice, the dual tree consists of two discrete Wavelet transforms working in parallel. The branches of the tree are interpreted as the real part, respectively, the imaginary one of a complex wavelet. Thus the complex wavelet transform is obtained.

The continuous Wavelet transform $(W_\psi f)(s,\tau)$, of the signal $f(t) \in L^2(R)$ can be determined as:

$$(W_\psi f)(s,\tau)=\int_{-\infty}^{+\infty} f(t)\cdot\frac{1}{\sqrt{s}}\cdot\overline{\psi\left(\frac{t-\tau}{s}\right)}\cdot dt \qquad (1)$$

where, $\overline{\psi}$ denote the complex conjugate of ψ .

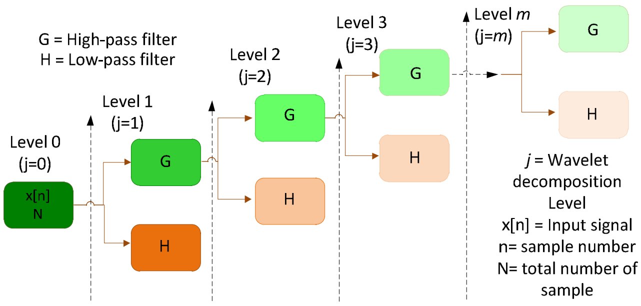

Fig.1- Wavelet decomposition tree

A wavelet filter acts as an averaging filter or a filter that detects details when a signal is decomposed with wavelets. A part of the consequent wavelet coefficients match with details in the data set. The detail significance is proportional with the amplitude of the waves - if they are small, they can be left out, without essentially influencing the data set main features [7]. The main idea of thresholding is to set all coefficients that are below a specific threshold at zero value. In order to rebuild the data set these coefficients are utilized in an inverse wavelet transform [8]. Deficiencies and design issues of such decomposition lead to the development of new processing methods.

We decided to propose a new method for estimating the optimal level of decomposing for the wavelet filter.

We are introducing a new time frequency approach, an extension of the Partial Directed Coherence (*PDC)* method, to assess coupling dynamics information in multivariate dynamic systems [9].

PDC approach is able to detect direct and indirect couplings between two time series. *PDC* is based on an *m*-dimensional multichannel autoregressive model (*MAR)* and uses an *MAR* process with order *p*:

$$\begin{bmatrix} x_1(n) \\ \vdots \\ x_N(n) \end{bmatrix} = \sum_{r=1}^{p} A_r(n) \begin{bmatrix} x_1(n-r) \\ \vdots \\ x_N(n-r) \end{bmatrix} + \begin{bmatrix} W_1(n) \\ \vdots \\ W_N(n) \end{bmatrix} \qquad (2)$$

Where the w vector is the white noise and A_r matrices are expressed by means of the next formulation:

$$A_r(n) = \begin{bmatrix} a_{11}(r,n) & \cdots & a_{1N}(r,n) \\ \vdots & a_{ij}(r,n) & \vdots \\ a_{N1}(r,n) & \cdots & a_{NN}(r,n) \end{bmatrix} \qquad (3)$$

with *r=1,.., p* model order. a_{ij} parameters represent the linear interaction effect of $x_j\,(n-r)$ onto $x_i(n)$. They are estimated by means of an adaptive autoregressive approach [10], the main advantage of which is the possibility of analyzing time-varying signals by updating the calculations for each time sample under investigation.

By calculating the Fourier transform of $A_r(n)$ matrix, more specific by calculating *A(n, t)* coefficient matrix in frequency domain:

$$A(n,f) = I - \sum_{r=1}^{p} A_r(n) z^{-r}\big|_{z=e^{i2\pi f}} \qquad (4)$$

where I is the identity matrix, a number of time-varying measurements of connectivity can be established.

The *PDC* coupling estimation between two time series (*Xi* and *Xj*) was defined by Baccala et al. [11] as:

$$\pi_{ij}(n) \overset{\Delta}{=} \frac{A_{ij}(n,f)}{\sqrt{a_j^H(n,f)a_j(n,f)}} \qquad (5)$$

where $\pi_{ij}(n)$ is the correlation parameter, $(.)^H$ the Hermitian transpose, $A_{ij}(n,f)$ the $A_r(n)$,the Fourier transform of the matrix in the frequency domain, $a_j(n,f)$ the *j'th* column number of the matrix *A(n,f)*, *n* the number of windows and *f*, the frequency.

The π_{ij} parameter normalization conditions in the frequency domain ($\pi_{ij}(f)$) were defined as:

$$0 \le \left|\pi_{ij}(f)\right| \le 1, \quad \sum_{i=1}^{m} \left|\pi_{ij}(f)\right| = 1 \qquad (6)$$

for all $1 \le j \le m$ values.

These measures were considered to provide information on the presence and level of causal correlation between two time series (*Xi* and *Xj*) as follows:

a) high values reflecting a directionally linear influence from *Xj* to *Xi*, meaning that, for values equal to 1, all the causal influences originating from *Xj* are directed towards *Xi,*
b) low values (≈0) suggesting the absence of any causal correlation from *Xj* to *Xi*, meaning that *Xj* does not influence *Xi*.

In order to estimate the coupling level (*CL*) between two time series belonging to the same system and to estimate the optimal level of the wavelet filter, the calculation of a new parameter was proposed, by employing the following equations:

$$a = mean \;\; PDC(X_i \to X_j)$$
$$b = mean \;\; PDC(X_{i-1} \to X_{j-1})$$
$$CL = \begin{cases} WoptLvl = WactualLvl + 1, \; if \; a-b>0 \\ WoptLvl = WactualLvl \, , \; if \; a-b=0 \\ WoptLvl = WactualLvl - 1 \, , \; if \; a-b<0 \end{cases} \qquad (7)$$

where, $WactualLvl$ is the Wavelet's actual level and $WoptLvl$ is the Wavelet's optimal level of decomposition. These measures provide information on wavelet coefficient as follows: a) if the previous value is lower than the current value then the order of the wavelet decomposition is equal to the previous value plus 1. b) if the previous value is higher than or equal to the current value, then the optimal order of decomposition is equal to the previous value.

The main idea of the optimization algorithm is illustrated in Fig. 2, where a signal received from an accelerometer is processed and analyzed by using the Wavelet transform until an optimal level of decomposition is established and the useful signal is achieved [10]. This elementary structure was proposed and studied in order to obtain a further general tuning method for the inertial sensor denoising, with the wavelet method. In this new general structure, the reference signals will be provided by a GPS, while the disrupted input signals in PDC are the outputs of the inertial navigation system (INS) (Fig. 3).

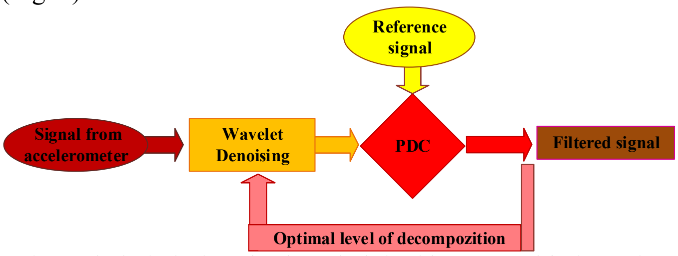

Fig.2 - The logical scheme/mathematical algorithm proposed for improving the signals acquired from an accelerometer

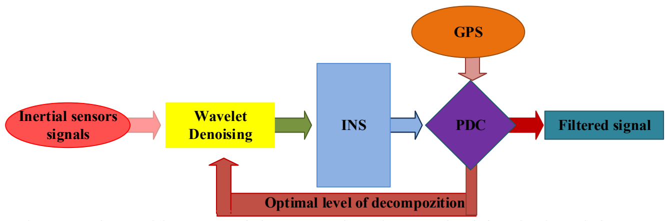

Fig. 3 – The architecture of the general tuning method for the inertial sensor denoising with wavelet method

For the here-presented study we simulated noisy and noiseless (clean) signals received from a miniaturized accelerometer in order to acknowledge an offline tuning of the wavelet function used in the denoising process of the accelerometer. The noiseless signals were used as reference signals (equivalent to the signals received from a GPS device) to correct the errors of the accelerometer. The noisy signals were correlated with the noiseless signals, by applying equation 7, in order to achieve the optimal level of decomposition of the wavelet filter leading to - after the accelerometer calibration - the achievement of more accurate acceleration data.

In simulations, a sinusoidal signal, generated by using the *wnoise Matlab* function for "Noisy wavelet test data", Fig. 4, was considered as a reference signal. This signal was corrupted by different types of noise (additive Gaussian white noise) as it may be seen in Fig. 5.

The method was implemented in Matlab for testing and validation after the mathematical problem was established.

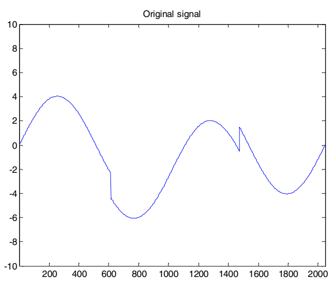

Fig 4 - Original signal

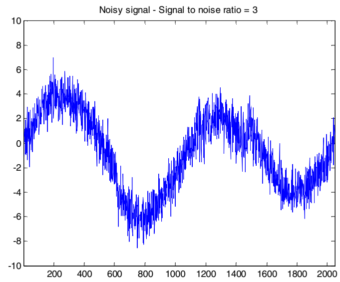

Fig. 5 - Signal corrupted with additive Gaussian white noise

III. Results and Discussions

By applying the proposed algorithm to the corrupted signal, using as reference the original signal (equivalent to the *GPS* signal), the $WoptLvl$ and *CL* achieved values were recorded in Table 1. According to equation 7, we can observe that the optimal level of decomposition is 10 for *CL* = 0.851520.

Table 1.

$WactualLvl$	CL
2	0.142700
3	0.289110
4	0.552780
5	0.795460
6	0.811420
7	0.811610
8	0.812420
9	0.835450
10	0.851520
11	0.811660
12	0.793500
13	0.778150
14	0.770220

For a visual comparison of the coupling level between the corrupted and the original signals, the coupling level diagrams for five different coupling levels were plotted by means of a short time implementation, with a window of 300 sample length, Figures 6 and 7. In both figures, *y*-axis represents the number of windows and the *x*-axis represents the normalized frequency between 0 and 1.

As it can be seen in all coupling diagrams, transitions from coupling to uncoupling, from strong level of couplings (represented in red color and shades of red) towards the absence of coupling (represented in blue color and shades of blue) are visible; an absence of coupling – a predominantly blue color is visible in Figure 6 suggesting that for $CL = 0.289110$, the investigated signals are uncorrelated or the level of correlation is very low and the achieved data corresponds in a limited propotion with the real data (the original signal). We are interested in achieving the highest level of coupling between the signals, *CL* values ≈ 1. A higher level of coupling can be seen in figure 8, for *CL* = 0.851520, where the predominantly red color suggests the presence of a strong level of coupling between the two signals. This level of coupling indicates that, for $CL = 0.851520$ $WactualLvl = 10$ is the $WoptLvl$.

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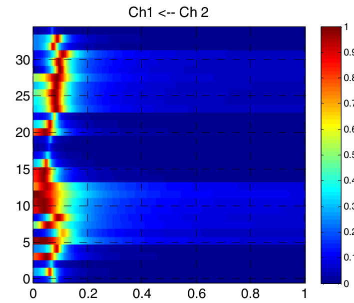

Fig 6 - Signal achieved for $WacualLvl$=3

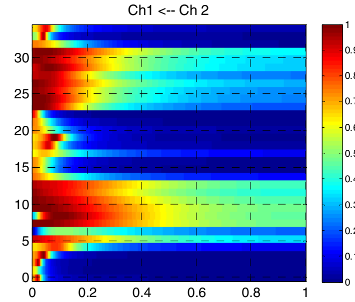

Fig. 7 – The coupling level diagram for $CL =$ 0.811420 , $WactualLvl = 6$

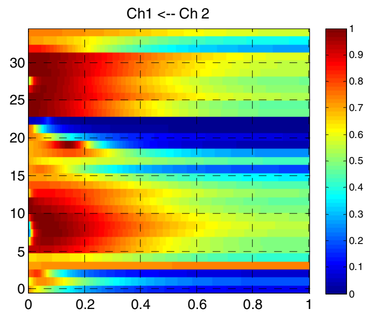

Fig. 8 – The coupling level diagram for $CL =$ 0.851520, for which $WactualLvl = WoptLvl = 10$

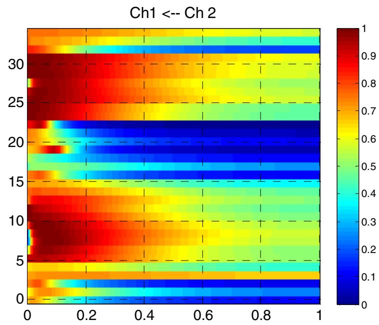

Fig. 9 - Signal achieved for $WactualLvl$=*14*

The sinusoidal signals achieved for different $WactualLvl$ were displayed in figures 10, 11, 12, 13 and 14.

After a careful visual inspection of figures 10 - 14 one can see that as the CL level increases, the signal achieved is more similar to the original signal but when/after CL = 14, the signal shape changes and turns into a sinusoidal signal, which loses the characteristics of the original signal. Also from the visual inspection we concluded that sinusoidal signal achieved for $WactualLvl$=10, which may be seen in Fig. 11 is/ was the optimum level of decomposition - $WoptLvl$.

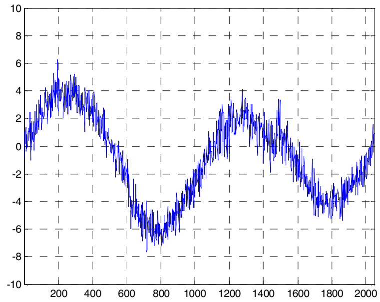

Fig. 10 - Signal achieved for $WactualLvl$=1

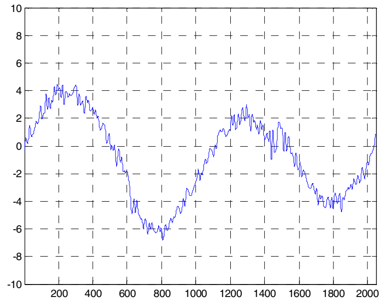

Fig. 11 - Signal achieved for $WactualLvl$=3

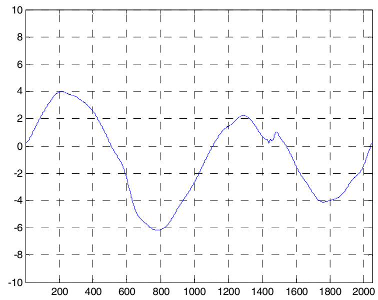

Fig. 12 - Signal achieved for $WactualLvl$ = 6

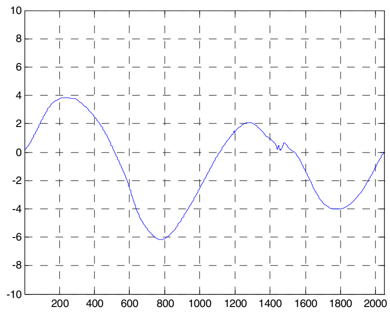

Fig. 13 - Signal achieved for $WactualLvl = WoptLvl$=10

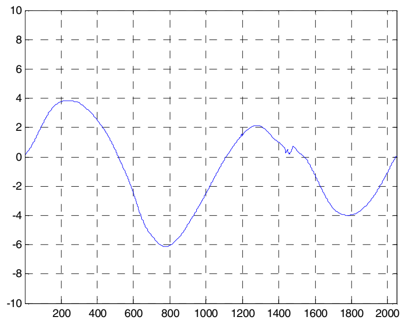

Fig. 14 - Signal achieved for $WactualLvl$=14

By using the proposed configuration from Figure 3, the correlation between useful signals, signals received from an accelerometer or an inertial measurement unit/*INS* can be tracked and corrected by using signals received from a GPS device (in a pre-calibration phase of the INS). This correlation becomes less clear when the signals achieved from the INS become correlated with the signals received from GPS, the correlation level reaches values close to 1, resulting in reduced errors of the navigation system (caused by the noise).

IV. CONCLUSIONS

This is a topical issue which brings significant improvement in the inertial navigation systems signal processing having a clear-cut role in positioning investigations.

The purpose of this research was to improve the performance of inertial navigation systems and their level of accuracy for situations when the GPS becomes unavailable. An improved version of the Wavelet filter was proposed for filtering/denoising the signals received from an accelerometer.

We intend to implement this algorithm for pre-calibrating a two-dimensional navigation system in the horizontal plan in order to improve its accuracy in positioning. By establishing the best coupling level of signals received from INS and GPS, using the GPS signal as reference, the optimal level of decomposition of the wavelet transform can be established and the proposed algorithm can be implemented in the inertial measurement unit as a real-time evaluation criterion with the purpose of achieving real-time data.

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