Dynamic response of a doubly curved shallow shell rectangular in plan impacted by a sphere

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Abstract—Large amplitude (geometrically non-linear) vibrations of doubly curved shallow shells with rectangular base under the low-velocity impact by an elastic sphere are investigated. It is assumed that the shell is simply supported and partial differential equations are obtained in terms of shell's transverse displacement and Airy's stress function. The local bearing of the shell and impactor's materials is neglected with respect to the shell deflection in the contact region. The equations of motion are reduced to a set of infinite nonlinear ordinary differential equations of the second order in time and with cubic and quadratic nonlinearities in terms of the generalized displacements. Assuming that only two natural modes of vibrations dominate during the process of impact and applying the method of multiple time scales, the set of equations is obtained, which allows one to find the time dependence of the contact force and to determine the contact duration and the maximal contact force.

Keywords—Doubly curved shallow shell rectangular in base, impact interaction, method of multiple time scales

I. Introduction

Doubly curved panels are widely used in aeronautics, aerospace and civil engineering and are subjected to dynamic loads that can cause vibration amplitude of the order of the shell thickness, giving rise to significant non-linear phenomena [1]–[4].

A review of the literature devoted to dynamic behaviour of curved panels and shells could be found in Amabili and Paidoussis [5], as well as in [3], wherein it has been emphasized that free vibrations of doubly curved shallow shells were studied in the majority of papers either utilizing a slightly modified version of the Donnell's theory taking into account the double curvature [1, 6] or the nonlinear first-order theory of shells [7, 8].

Large-amplitude vibrations of doubly curved shallow shells with rectangular base, simply supported at the four edges and subjected to harmonic excitation were investigated in [3], while chaotic vibrations were analyzed in [4]. It has been revealed that such an important nonlinear phenomenon as the occurrence of internal resonances in the problems considered in [3] and [4] is of fundamental importance in the study of curved shells.

In spite of the fact that the impact theory is substantially developed, there is a limited number of papers devoted to the problem of impact over geometrically nonlinear shells.

The nonlinear impact response of laminated composite cylindrical and doubly curved shells was analyzed using a modified Hertzian contact law in [9] via a finite element model, which was developed based on Sander's shell theory involving shear deformation effects and nonlinearity due to large deflection. A nine-node isoparametric quadrilateral element was used to model the curved shell. The nonlinear time dependent equations were solved using an iterative scheme and Newmark's method. Numerical results for the contact force and center deflection histories were presented for various impactor conditions, shell geometry and boundary conditions.

Later large deflection dynamic responses of laminated composite cylindrical shells under impact have been analyzed in [10] by the geometrically nonlinear finite element method based on a generalized Sander's shell theory with the first order shear deformation and the von Karman large deflection assumption.

Nonlinear dynamic response for shallow spherical moderate thick shells with damage under low velocity impact has been studied in [11] by using the orthogonal collocation point method and the Newmark method to discrete the unknown variable function in space and in time domain, respectively, and the whole problem is solved by the iterative method. Further this approach was generalized for investigating dynamic response of elasto-plastic laminated composite shallow spherical shell under low velocity impact [12] and nonlinear dynamic response for functionally graded shallow spherical shell under low velocity impact in thermal environment [13].

The nonlinear transient response of laminated composite shell panels subjected to low velocity impact in hygrothermal environments was investigated in [14] using finite element method considering doubly curved thick shells involving large deformations with Green-Lagrange strains. The analysis was carried out using quadratic eight-node isoparametric element.

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A modified Hertzian contact law was incorporated into the finite element program to evaluate the impact force. The nonlinear equation was solved using the Newmark average acceleration method in conjunction with an incremental modified Newton-Raphson scheme. A parametric study was carried out to investigate the effects of the curvature and side to thickness ratios of simply supported composite cylindrical and spherical shell panels.

The impact behaviour and the impact-induced damage in laminated composite cylindrical shell subjected to transverse impact by a foreign object were studied in [15] using three-dimensional non-linear transient dynamic finite element formulation. Non-linear system of equations resulting from non-linear strain displacement relation and non-linear contact loading was solved using the Newton-Raphson incremental-iterative method. Some example problems of graphite/epoxy cylindrical shell panels were considered with variation of impactor and laminate parameters and influence of geometrical non-linear effect on the impact response and the resulting damage was investigated.

In the present paper, a new approach is proposed for the analysis of the impact interactions of nonlinear doubly curved shallow shells with rectangular base under the low-velocity impact by an elastic sphere. It is assumed that the shell is simply supported and partial differential equations are obtained in terms of shell's transverse displacement and Airy's stress function. The local bearing of the shell and impactor's materials is neglected with respect to the shell deflection in the contact region. The equations of motion are reduced to a set of infinite nonlinear ordinary differential equations of the second order in time and with cubic and quadratic nonlinearities in terms of the generalized displacements.

Assuming that only two natural modes of vibrations dominate during the process of impact and applying the method of multiple time scales [16], the set of dynamic equations is obtained, which allows one to find the time dependence of the contact force and to determine the contact duration and the maximal contact force.

II. PROBLEM FORMULATION AND GOVERNING EQUATIONS

Assume that an elastic or rigid sphere of mass M moves along the z-axis towards a thin walled doubly curved shell with thickness h, curvilinear lengths a and b, principle curvatures k_x and k_y and rectangular base, as shown in Fig. 1. Impact occurs at the moment t=0 with the velocity εV_0 (ε is a small value) at the point N with Cartesian coordinates x_0, y_0.

According to Donnell's nonlinear shallow shell theory, the equations of motion could be obtained in terms of lateral deflection w and Airy's stress function φ [17]

$$\frac{D}{h}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \phi}{\partial x^2}$$
$$-2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 \phi}{\partial x \partial y}+k_y\frac{\partial^2 \phi}{\partial x^2}+k_x\frac{\partial^2 \phi}{\partial y^2}+\frac{F}{h}-\rho\ddot{w}, \tag{1}$$

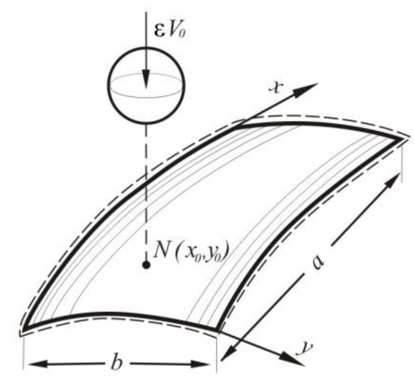

Fig. 1 Geometry of the doubly curved shallow shell

$$\frac{1}{E}\left(\frac{\partial^4 \phi}{\partial x^4}+2\frac{\partial^4 \phi}{\partial x^2 \partial y^2}+\frac{\partial^4 \phi}{\partial y^4}\right)=-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2$$
$$-k_y\frac{\partial^2 w}{\partial x^2}-k_x\frac{\partial^2 w}{\partial y^2}, \tag{2}$$

where $D=\dfrac{Eh^3}{12(1-\nu^2)}$ is the cylindrical rigidity, ρ is the density, E and ν are the elastic modulus and Poisson's ratio, respectively, t is time, $F=P(t)\delta(x-x_0)\delta(y-y_0)$ is the contact force, $P(t)$ is yet unknown function, δ is the Dirac delta function, x and y are Cartesian coordinates, overdots denote time-derivatives, $\phi(x,y)$ is the stress function which is the potential of the in-plane force resultants

$$N_x=h\frac{\partial^2 \phi}{\partial y^2},\quad N_y=h\frac{\partial^2 \phi}{\partial x^2},\quad N_{xy}=-h\frac{\partial^2 \phi}{\partial x \partial y}. \tag{3}$$

The equation of motion of the sphere is written as

$$M\ddot{z}=-P(t) \tag{4}$$

subjected to the initial conditions

$$z(0)=0,\quad \dot{z}(0)=\varepsilon V_0, \tag{5}$$

where $z(t)$ is the displacement of the sphere, in so doing

$$z(t)=w(x_0,y_0,t). \tag{6}$$

Considering a simply supported shell with movable edges, the following conditions should be imposed at each edge:

$$w=0,\quad \int_0^b N_{xy}dy=0,\quad N_x=0,\quad M_x=0,\quad \text{at}\ \ x=0,a, \tag{7}$$

$$w=0,\quad \int_0^a N_{xy}dx=0,\quad N_y=0,\quad M_y=0,\quad \text{at}\ \ y=0,b, \tag{8}$$

where M_x and M_y are the moment resultants.

The suitable trial function that satisfies the geometric boundary conditions is

$$w(x,y,t)=\sum_{p=1}^{\tilde p}\sum_{q=1}^{\tilde q}\xi_{pq}(t)\sin\left(\frac{p\pi x}{a}\right)\sin\left(\frac{q\pi y}{b}\right), \qquad (9)$$

where p and q are the number of half-waves in x and y directions, respectively, and $\xi_{pq}(t)$ are the generalized coordinates. Moreover, $\tilde p$ and $\tilde q$ are integers indicating the number of terms in the expansion.

Substituting (9) in (6) and using (4), we obtain

$$P(t)=-M\sum_{p=1}^{\tilde p}\sum_{q=1}^{\tilde q}\ddot\xi_{pq}(t)\sin\left(\frac{p\pi x_0}{a}\right)\sin\left(\frac{q\pi y_0}{b}\right). \qquad (10)$$

In order to find the solution of the set of equations (1) and (2), it is necessary first to obtain the solution of (2). For this purpose, let us substitute (9) in the right-hand side of (2) and seek the solution of the equation obtained in the form

$$\phi(x,y,t)=\sum_{m=1}^{\tilde m}\sum_{n=1}^{\tilde n}A_{mn}(t)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \qquad (11)$$

where $A_{mn}(t)$ are yet unknown functions.

Substituting (9) and (11) in (2) and using the orthogonality conditions of sines within the segments $0\le x\le a$ and $0\le y\le b$, we have

$$\begin{aligned}A_{mn}(t)&=\frac{E}{\pi^2}K_{mn}\xi_{mn}(t)\\&+\frac{4E}{a^3b^3}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^{-2}\sum_k\sum_l\sum_p\sum_q B_{pqklmn}\xi_{pq}(t)\xi_{kl}(t),\end{aligned} \qquad (12)$$

where

$$\begin{aligned}B_{pqklmn}&=pqklB^{(2)}_{pqklmn}-p^2l^2B^{(1)}_{pqklmn},\\ B^{(1)}_{pqklmn}&=\int_0^a\int_0^b\sin\left(\frac{p\pi x}{a}\right)\sin\left(\frac{q\pi y}{b}\right)\sin\left(\frac{k\pi x}{a}\right)\\&\times\sin\left(\frac{l\pi y}{b}\right)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)dxdy,\\ B^{(2)}_{pqklmn}&=\int_0^a\int_0^b\cos\left(\frac{p\pi x}{a}\right)\cos\left(\frac{q\pi y}{b}\right)\cos\left(\frac{k\pi x}{a}\right)\\&\times\cos\left(\frac{l\pi y}{b}\right)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)dxdy,\\ K_{mn}&=\left(k_y\frac{m^2}{a^2}+k_x\frac{n^2}{b^2}\right)^2\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^{-2}.\end{aligned} \qquad (13)$$

Substituting then (9)-(12) in (1) and using the orthogonality condition of sines within the segments $0\le x\le a$ and $0\le y\le b$, we obtain an infinite set of coupled nonlinear ordinary differential equations of the second order in time for defining the generalized coordinates

$$\begin{aligned}&\ddot\xi_{mn}(t)+\Omega_{mn}^2\xi_{mn}(t)\\&+\frac{8\pi^2E}{a^3b^3\rho}\sum_p\sum_q\sum_k\sum_l B_{pqklmn}\left(K_{kl}-\frac{1}{2}K_{mn}\right)\xi_{pq}(t)\xi_{kl}(t)\\&+\frac{32\pi^4E}{a^6b^6\rho}\sum_r\sum_s\sum_i\sum_j\sum_k\sum_l\sum_p\sum_q B_{rsijmn}B_{pqklij}\xi_{rs}(t)\xi_{pq}(t)\xi_{kl}(t)\\&+\frac{4M}{ab\rho h}\sin\left(\frac{m\pi x_0}{a}\right)\sin\left(\frac{n\pi y_0}{b}\right)\\&\times\sum_p\sum_q\ddot\xi_{pq}(t)\sin\left(\frac{p\pi x_0}{a}\right)\sin\left(\frac{q\pi y_0}{b}\right)=0,\end{aligned} \qquad (14)$$

where Ω_{mn}^2 are natural frequencies of the target defined as

$$\Omega_{mn}^2=\frac{E}{\rho}\left[\frac{\pi^4h^2}{12(1-\nu^2)}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2+K_{mn}\right]. \qquad (15)$$

The last term in each equation from (14) describes the influence of the coupled impact interaction of the target with the impactor of the mass M applied at the point with the coordinates x_0, y_0.

In order to study this additional nonlinear phenomenon induced by the coupled impact interaction, we suppose that only two natural modes of vibrations are excited during the process of impact, namely, $\Omega_{\alpha\beta}$ and $\Omega_{\gamma\delta}$. Then the set of equations (14) is reduced to the following two equations written in the dimensionless form:

$$\begin{aligned}&p_{11}\ddot\zeta_1+p_{12}\ddot\zeta_2+\zeta_1\Omega_1^2+p_{13}\zeta_1^2+p_{14}\zeta_2^2+p_{15}\zeta_1\zeta_2\\&\qquad+p_{16}\zeta_1^3+p_{17}\zeta_1\zeta_2^2=0,\end{aligned} \qquad (16)$$

$$\begin{aligned}&p_{21}\ddot\zeta_1+p_{22}\ddot\zeta_2+\zeta_2\Omega_2^2+p_{23}\zeta_2^2+p_{24}\zeta_1^2+p_{25}\zeta_1\zeta_2\\&\qquad+p_{26}\zeta_2^3+p_{27}\zeta_1^2\zeta_2=0,\end{aligned} \qquad (17)$$

where $\zeta_1=\frac{\xi_{\alpha\beta}}{a}$, $\zeta_2=\frac{\xi_{\lambda\delta}}{a}$, $\Omega_1=\Omega^*_{\alpha\beta}$ and $\Omega_2=\Omega^*_{\gamma\delta}$ are dimensionless natural frequencies

$\Omega^{*2}_{mn}=\dfrac{\pi^4h^2}{12(1-\nu^2)a^2}\left(m^2+n^2\dfrac{a^2}{b^2}\right)^2+a^4K_{mn}$, dimensionless coefficients p_{ij} (i=1, 2; j=1, 2,…,7) could be easily obtained from (14) using (13), wherein $x^*=x/a$, $y^*=y/b$, and $t^*=\dfrac{t}{a}\sqrt{\dfrac{E}{\rho}}$.

III. Method of Solution

In order to solve a set of two nonlinear equations (16) and (17), we apply the method of multiple time scales [16] for constructing the solution of Eqs. (13)

$$\zeta_1(t)=\varepsilon X^1_{\alpha\beta}(T_0,T_1)+\varepsilon^2X^2_{\alpha\beta}(T_0,T_1), \qquad (18)$$

$$\zeta_2(t)=\varepsilon X^1_{\gamma\delta}(T_0,T_1)+\varepsilon^2X^2_{\gamma\delta}(T_0,T_1), \qquad (19)$$

where $T_n=\varepsilon^nt$ are new independent variables, among them: $T_0=t$ is a fast scale characterizing motions with the natural frequencies, and $T_1=\varepsilon t$ is a slow scale characterizing the

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modulation of the amplitudes and phases of the modes with nonlinearity.

Considering that

$$\frac{d^2}{dt^2}\zeta_i = \varepsilon(D_0^2 X_{ij}^1) + \varepsilon^2(D_0^2 X_{ij}^2 + 2D_0 D_1 X_{ij}^1),$$

where $ij = \alpha\beta$ or $\gamma\delta$, and $D_i^n = \partial^n / \partial T_i^n$ ($n = 1,2,\ i = 0,1$), and substituting the proposed solution (18) and (19) in (16) and (17), after equating the coefficients at like powers of ε to zero, we are led to a set of recurrence equations to various orders:

to order ε

$$p_{11} D_0^2 X_1^1 + p_{12} D_0^2 X_2^1 + \Omega_1^2 X_1^1 = 0\,, \tag{20}$$

$$p_{21} D_0^2 X_1^1 + p_{22} D_0^2 X_2^1 + \Omega_2^2 X_2^1 = 0\,; \tag{21}$$

to order ε^2

$$\begin{aligned} p_{11} D_0^2 X_1^2 + p_{12} D_0^2 X_2^2 + \Omega_1^2 X_1^2 &= -2 p_{11} D_0 D_1 X_1^1 \\ &- 2 p_{12} D_0 D_1 X_2^1 - p_{13}(X_1^1)^2 - p_{14}(X_2^1)^2 - p_{15} X_1^1 X_2^1, \end{aligned} \tag{22}$$

$$\begin{aligned} p_{21} D_0^2 X_1^2 + p_{22} D_0^2 X_2^2 + \Omega_2^2 X_2^2 &= -2 p_{21} D_0 D_1 X_1^1 \\ &- 2 p_{22} D_0 D_1 X_2^1 - p_{23}(X_1^1)^2 - p_{24}(X_2^1)^2 - p_{25} X_1^1 X_2^1, \end{aligned} \tag{23}$$

where for simplicity is it denoted $X_1^1 = X_{\alpha\beta}^1$, $X_2^1 = X_{\gamma\delta}^1$, $X_1^2 = X_{\alpha\beta}^2$, and $X_2^2 = X_{\gamma\delta}^2$.

A. *Solution of Equations at Order of ε*

Following Rossikhin and Shitikova [21], we seek the solution of (20) and (21) in the form:

$$X_1^1 = A_1(T_1)\, e^{i\omega_1 T_0} + A_2(T_1) e^{i\omega_2 T_0} + \mathrm{cc}\,, \tag{24}$$

$$X_2^1 = \alpha_1 A_1(T_1)\, e^{i\omega_1 T_0} + \alpha_2 A_2(T_1) e^{i\omega_2 T_0} + \mathrm{cc}\,, \tag{25}$$

where $A_1(T_1)$ and $A_2(T_1)$ are unknown complex functions, cc is the complex conjugate part to the preceding terms, and $\bar{A}_1(T_1)$ and $\bar{A}_2(T_1)$ are their complex conjugates,

$$\begin{gathered} \omega_{1,2}^2 = \frac{(p_{22}\Omega_1^2 + p_{11}\Omega_2^2) \pm \sqrt{(p_{22}\Omega_1^2 - p_{11}\Omega_2^2)^2 + 4\,\Omega_1^2\Omega_2^2 p_{12} p_{21}}}{2(p_{11}p_{22} - p_{12}p_{21})}\,, \\ \alpha_1 = -\frac{p_{11}\omega_1^2 - \Omega_1^2}{p_{12}\omega_1^2} = -\frac{p_{21}\omega_1^2}{p_{22}\omega_1^2 - \Omega_2^2} \\ \alpha_2 = -\frac{p_{11}\omega_2^2 - \Omega_1^2}{p_{12}\omega_2^2} = -\frac{p_{21}\omega_2^2}{p_{22}\omega_2^2 - \Omega_2^2}\,, \\ p_{11} = 1 + d_1\,, \qquad p_{22} = 1 + d_2\,, \qquad p_{12} = p_{21} = \frac{4M}{\rho hab} s_1 s_2, \\ d_1 = \frac{4M}{\rho hab} s_1^2, \qquad d_2 = \frac{4M}{\rho hab} s_2^2, \\ s_1 = \sin(\alpha\pi x_0^*)\sin(\beta\pi y_0^*), \qquad s_2 = \sin(\gamma\pi x_0^*)\sin(\delta\pi y_0^*)\,. \end{gathered} \tag{26}$$

Reference to relationships (26) shows that ω_1 and ω_2 are the frequencies of the coupled process of impact interaction of the impactor and the target. As the impactor mass $M \to 0$, the frequencies ω_1 and ω_2 tend to the natural frequencies of the shell vibrations Ω_1 and Ω_2, respectively. Coefficients s_1 and s_2 depend on the numbers of the natural modes involved in the process of impact interaction, $\alpha\beta$ and $\gamma\delta$, and on the coordinates of the contact force application x_0^*, y_0^*, resulting in the fact that their particular combinations could vanish coefficients s_1 and s_2 and, thus, coefficients $p_{12} = p_{21}$.

B. *Solution of Equations at Order of ε²*

Substituting (24) and (25) in (22) and (23), we obtain

$$\begin{aligned} p_{11} D_0^2 X_1^2 + p_{12} D_0^2 X_2^2 + \Omega_1^2 X_1^2 &= -2i\omega_1 (p_{11} + \alpha_1 p_{12}) e^{i\omega_1 T_0} D_1 A_1 \\ &- 2i\omega_2 (p_{11} + \alpha_2 p_{12}) e^{i\omega_2 T_0} D_1 A_2 \\ &- (p_{13} + \alpha_1^2 p_{14} + \alpha_1 p_{15}) A_1 \left[A_1 e^{2i\omega_1 T_0} + \bar{A}_1 \right] \\ &- (p_{13} + \alpha_2^2 p_{14} + \alpha_2 p_{15}) A_2 \left[A_2 e^{2i\omega_2 T_0} + \bar{A}_2 \right] \\ &- 2\left[p_{13} + \alpha_1\alpha_2 p_{14} + (\alpha_1 + \alpha_2) p_{15} \right] A_1 \\ &\times \left[A_2 e^{i(\omega_1+\omega_2)T_0} + \bar{A}_2 e^{i(\omega_1-\omega_2)T_0} \right] + \mathrm{cc}, \end{aligned} \tag{27}$$

$$\begin{aligned} p_{21} D_0^2 X_1^2 + p_{22} D_0^2 X_2^2 + \Omega_2^2 X_2^2 &= -2i\omega_1 (p_{21} + \alpha_1 p_{22}) e^{i\omega_1 T_0} D_1 A_1 \\ &- 2i\omega_2 (p_{21} + \alpha_2 p_{22}) e^{i\omega_2 T_0} D_1 A_2 \\ &- (p_{23} + \alpha_1^2 p_{24} + \alpha_1 p_{25}) A_1 \left[A_1 e^{2i\omega_1 T_0} + \bar{A}_1 \right] \\ &- (p_{23} + \alpha_2^2 p_{24} + \alpha_2 p_{25}) A_2 \left[A_2 e^{2i\omega_2 T_0} + \bar{A}_2 \right] \\ &- 2\left[p_{23} + \alpha_1\alpha_2 p_{24} + (\alpha_1 + \alpha_2) p_{25} \right] A_1 \\ &\times \left[A_2 e^{i(\omega_1+\omega_2)T_0} + \bar{A}_2 e^{i(\omega_1-\omega_2)T_0} \right] + \mathrm{cc}. \end{aligned} \tag{28}$$

For the obtained set of coupled equations (27) and (28) all terms proportional to $e^{i\omega_1 T_0}$ and $e^{i\omega_2 T_0}$ are circular terms, so they should be eliminated from the further solution.

Thus, we obtain the following conditions of solvability:

$$D_1 A_1 = 0\,, \qquad D_1 A_2 = 0\,, \tag{29}$$

whence it follows that the functions A_1 and A_2 are T_1-independent.

C. *Determination of the Contact Force*

Representing A_1 and A_2 in the polar form

$$A_i = a_i e^{i\varphi_i} \quad (i = 1,2), \tag{30}$$

relationships (24) and (25) take the form

$$X_1^1 = 2a_1(0)\cos[\omega_1 t + \varphi_1(0)] + 2a_2(0)\cos[\omega_2 t + \varphi_2(0)], \tag{31}$$

$$X_2^1 = 2\alpha_1 a_1(0)\cos[\omega_1 t + \varphi_1(0)] + 2\alpha_2 a_2(0)\cos[\omega_2 t + \varphi_2(0)]\,, \tag{32}$$

wherein the initial amplitudes $a_i(0)$ and phases $\varphi_i(0)$ should be determined from the initial conditions.

Considering (31) and (32), the solution for the shell deflection (9) at the point of impact and the contact force (10) is the following:

$$w(x_0, y_0, t) = \varepsilon(X_1^1 s_1 + X_2^1 s_2)\,, \tag{33}$$

$$P(t) = -\varepsilon M (\ddot{X}_1^1 s_1 + \ddot{X}_2^1 s_2)\,. \tag{34}$$

IV. Conclusion

The procedure proposed in the present paper allows one to investigate the dynamic response of a nonlinear doubly curved shallow shell impacted by a sphere, to find the time dependence of the contact force and to determine the contact duration and the maximal contact force.

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