# Sliding mode control with Bond graph modeling applied on a robot leg

Alexandru I. Gal, Luige Vladareanu\*

**Abstract**— In this paper we use the bond graph modeling to control a mobile walking robots' leg. The legs' structure presents 2DOF for an easy understanding of the control laws' behavior. Along with this approach we used the sliding mode control method, which is a dynamic position control method, to achieve the reference tracking control. Also, a fuzzy control law was added to the main feedback loop, to improve the tracking speed and to lower the time needed by the legs' foot to reach the desired reference position. Compared to others, we achieved to eliminate override, a better time in reaching the desired position and a lower error rate..

*Keywords*—Bond Graph, Dynamic Control, Fuzzy Control, Sliding Mode Control.

### I. INTRODUCTION

A functional representation of a system is a representation that describes how the system works, or how it should work. This representation can be used in simulating and verifying the system, and to generate diagnostics by using many simulation environments like Matlab Simulink or Bond Graph modeling[1,2].

Robots are well-known as nonlinear systems that include a strong coupling between their dynamics (Craig, 1996). These characteristics along with structured and unstructured uncertainties cause by model imprecision and un-modeled dynamics make the motion control a difficult problem (Spong & Vidiasagar, 1989) that can be reduced by using the bond graph modeling approach. Modeling a system is based on its decomposition. This is why the solution of the majority of complex problems consists in modeling. A model simplifies the problem by abstracting certain subsets of its observable attributes. Thus, we can emphasis on the problems' relevant points and we can exclude for the respective problem the irrelevant ones [3, 5, 13].

A well-known approach, which was made to model the interaction between physical systems, is the bond graph method, designed by Henry Paynter in 1959 and Damic and

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Montgomery in 2003[5]. This method uses the analogy effortflow to describe the physical processes. These processes are graphical represented like elementary components with one or multiple ports. These ports represent the places where the systems interact with each other [7, 8]. By using this type of modeling allowed by the bond graphs, we made a virtual system that is a model of, and simulates, a walking robots' leg with 2 degrees of freedom. This was conducted in the Bond Graph simulation environment 20-Sim. By using this virtual system we could test a dynamic control law based on Sliding Control Method [2]. This type of simulation represents a new method of approaching the design and control of walking robots that are controller through a dynamic control method. The Sliding Mode Control (SMC) is used because its dynamic behavior can be modified according to specific options [2, 12] and because its closed loop response is undisturbed for a certain group of uncertainties (Lin et. all, 1998 [6]).

In this paper, we'll present a modified dynamic control SMC, developed with the help of bond graphs and for which we used a fuzzy control law to amplify the command signal for each joint/motor. The purpose of this paper is to improve a walking robot movement control on unstructured and bumped surfaces and to achieve improved tracking performances of position reference.

II. THE DYNAMIC CONTROL USING SLIDING CONTROL METHOD For the control system used, we have the main dynamic relation below:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + \tau_{d} = \tau \tag{1}$$

where, H is the inertial matrix, the vectors q,  $\dot{q}$ ,  $\ddot{q}$  are the position, speed and angular acceleration within the robots joints. The matrix C represents the Coriolis and centrifugal forces, and G is the gravitational vector. Also,  $\tau_d$  represents the vector of disturbances and the dynamics that are not computed, and  $\tau$  is the desired torque. Knowing that the system error is:

$$e = q_d - q \tag{2}$$

we chose the sliding surface as given in relation (3) (as shown by S.E. Shafiei in [4]). This value of s tends to 0 when the error e also tends to 0, meaning that  $\lim(s)=0$  when e>0.

By using the relation (3) we can rewrite the equation (1) and obtain equation (4), by replacing the value of q with the sum

q+s, which will have the value of the desired value of qd when s equals zero. The value of q becomes qd because we need to calculate the torque of a desired angular position.

$$s = \dot{e} + \lambda_1 e + \lambda_2 \int_0^t e dt \tag{3}$$

$$H(q)(\ddot{q}+\dot{s})+C(q,\dot{q})(\dot{q}+s)+G(q)+\tau_{d}=\tau \tag{4}$$

By replacing the terms of equations (3) and (4) we get:

$$H\dot{s} = -Cs + f + \tau_d - \tau \tag{5}$$

Results, that we can compute the torque:

$$\tau = \hat{f} + K_v s + K \cdot sat(s) \tag{6}$$

where,

$$\hat{f} = \hat{H} \left( \ddot{q}_d + \lambda_1 \dot{e} + \lambda_2 e \right) + \hat{C} \left( \dot{q}_d + \lambda_1 e + \lambda_2 \int_0^t e dt \right) + \hat{G}$$
 (7)

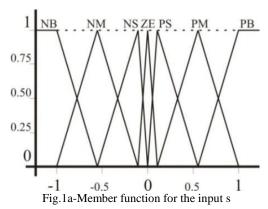
Equation (6) presents the force estimation, and s is the exterior PID tracking loop, Kv, K are positive diagonal matrix and are built so that the stability conditions are fulfilled, and  $\hat{f}$  is an estimation of f. The lambda 1 and 2 matrices were adjusted to get the best results. The sat(s) function used is the saturation function, in which  $\phi$  is a constant that defines an area around the sliding surface in which the control parameter K is lowered proportional to the constant  $\phi$ :

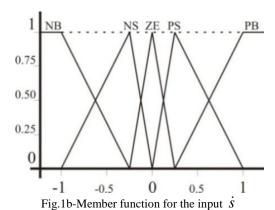
$$sat\left(\frac{s}{\varphi}\right) = \begin{cases} -1, & s \le -\varphi \\ \frac{s}{\varphi}, & -\varphi < s < \varphi \\ 1, & \varphi \le s \end{cases}$$
 (8)

$$K = N \cdot K_{fuzzy}, K_{V} = N_{V} \cdot K_{fuzzy}$$
 (9)

To calculate the two matrices K and Kv we use a fuzzy control law and two constant matrices, N and Nv. To calculate the  $K_{fuzzy}$  gain matrix [7, 11], we need two inputs, s and  $\dot{s}$  for which we have the membership functions in figure 1a and figure 1b. The abbreviation from figure 1a and figure 1b stands for: N=Negative, Z=Zero, P=Positive, V=Very, B=Big, M=Medium, S=Small. The control diagram used in the Sliding

Control Simulation is the one presented in figure 2, in which we can observe two main parts: the Sliding Control area and the Internal Motors control for the two controlled joints. The diagram has three function blocks that can be attributed to the sliding control method. The first block is the one that computes the value of  $\hat{f}$  from the relation (7). This block uses the values that were computed by many blocks: the reference generation blocks for each joint; the error computation bock; the function block that computes the dynamic matrix named "Calculate H, C, G".





The second block called *Sliding Surface* that forms the Sliding Mode Control Method is the block that computes the value of *s* according to relation (3) and it contains the PID computation of the error *e*, relation (2).

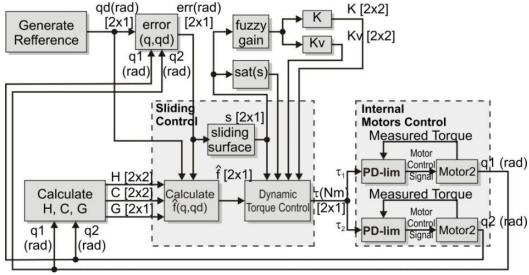


Fig. 2 – The control diagram of the Dynamic Leg Control using Sliding Control

Using these control blocks, we have made an internal torque control feedback loop and an external joint position feedback loop control, by using a dynamic approach along with the Sliding Control Method and the Fuzzy adjustment law.

### III. CASE STUDY ACCOMPLISHED BY USING BOND GRAPHS

Bond graphs are mainly used in modeling different mechatronic/electrical/hydraulically systems because this component part of the system is the most difficult to simulate. Thus, we could simulate the behavior of a joint's motor used in figure 3 to develop the robot joint system. The control system was applied to a walking robot leg that has 2 degrees of freedom with rotating joints. This system is presented in figure 2 [7, 9]. The kinematic structure from figure 4 corresponds to the equations system in relation (10).

$$\begin{bmatrix} x_{\scriptscriptstyle M} \\ y_{\scriptscriptstyle M} \end{bmatrix} = \begin{bmatrix} l_{\scriptscriptstyle 2} \sin \theta_{\scriptscriptstyle 1} + l_{\scriptscriptstyle 3} \sin \theta_{\scriptscriptstyle 2} \\ l_{\scriptscriptstyle 1} + l_{\scriptscriptstyle 2} \cos \theta_{\scriptscriptstyle 1} + l_{\scriptscriptstyle 3} \cos \theta_{\scriptscriptstyle 2} \end{bmatrix}$$
(10)

Also, we need the numerical values of the physical characteristics of the robot to test the control system.

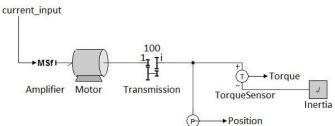
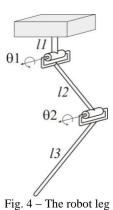


Fig. 3 – Bond Graph of the Robot Joint System

Thus, in Table 1, these values are presented and are chosen closer to reality [2].



structure

Table 1		
Parameter	Value	Type
m1	0,04	Kg
m2	0,44	Kg
m3	0,22	Kg
11	0,1	m
12	0,5	m
13	0,7	m
g	9,8	$m/s^2$

In order to implement the sliding mode control method we need to define the dynamic parameters that are used in relation (7), where H is the inertial matrix, C is the matrix of Coriolis effect and centrifugal forces, and G is the matrix of gravity tensor:

$$H(q) = \begin{bmatrix} (m_2 + m_3)l_2^2 + m_3l_3^2 + 2m_3l_2l_3\cos q_2 & m_3l_3^2 + m_3l_2l_3\cos q_2 \\ m_3l_3^2 + m_3l_2l_3\cos q_2 & m_3l_3^2 \end{bmatrix}$$
(11)

$$C(q, \dot{q}) = \begin{bmatrix} -m_3 l_2 l_3 \dot{q}_2 \sin q_2 & -m_3 l_2 l_3 (\dot{q}_1 + \dot{q}_2) \sin q_2 \\ m_3 l_2 l_3 \dot{q}_1 \sin q_2 & 0 \end{bmatrix}$$
(12)

$$G(q) = \begin{bmatrix} (m_2 + m_3)l_2^2 \frac{g}{l_2} \cos q_1 + m_3 l_2 l_3 \frac{g}{l_2} \cos(q_1 + q_2) \\ m_3 l_2 l_3 \frac{g}{l_2} \cos(q_1 + q_2) \end{bmatrix}$$
(13)

-1.5 <= NS0 < PS1.5<PM  $-3 \le NM$ Z=03<PB NB < -3NM < -1.5NS<0 PS <= 1.5PM <= 3Ś NB < -20400 250 400 150 50 15 50 NS = -10250 400 150 50 15 50 150 Z=0250 150 50 15 50 150 250 PS = 10150 50 15 50 150 250 400 PB> 20 50 15 50 150 250 400 400

Table 2 - Fuzzy rule for adjusting Kfuzzy

The presented control method uses a fuzzy logic gain to better control the approach of the robot foot to the sliding surface. The values of the fuzzy membership functions (figures 1a and 1b) are presented in table 2, values which were found by trial and error.

Also, we need to define the constants that were used in the sliding mode control combined with the fuzzy logic [4]. Besides the adjustments of parameters to achieve a better positioning error, the main variation from the classic SMC is the fuzzy control. In order to reduce the positioning error, we added a constraint to the sliding parameter s. This constraint removes the integral value from the sliding parameter calculation when its value is below a certain threshold.

By using all of these parameters and functions, we have made an internal torque control feedback loop and an external joint position feedback loop control. Also, by using a dynamic approach along with the Sliding Control Method and the Fuzzy adjustment law, through the bond graph simulation method, we could achieve a better control law.

$$s = \begin{cases} \dot{e} + \lambda_{1}e + \lambda_{2} \int edt, & \int edt < threshold \\ 0 & 0 \\ \dot{e} + \lambda_{1}e, & \int edt \ge threshold \\ 0 & 0 \end{cases}$$

$$(14)$$

## IV. RESULTS AND CONCLUSIONS

The control system was made and simulated with the help of the 20-Sim modeling environment which has the possibility of using bond graphs. To test the control system, we chose 2 sinusoidal signals of 2 and 2.5 radians amplitude.

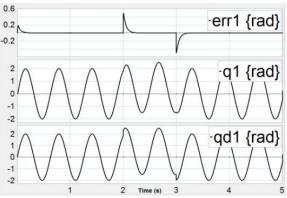


Fig. 5 – Joint 1: a) Error, b) Tracking Position, c)
Reference Position

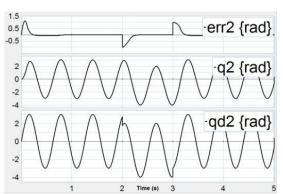


Fig. 6 – Joint 2: a) Error, b) Tracking Position, c)
Reference Position

In figure 5 and figure 6 we presented the obtained results according to the simulation. In these two figures, the bottom diagram represents the angular reference for each joint (figure 5 and respectively figure 6). The middle diagrams represent the system tracking signal, and the top diagram has the angular error signal for the two joint motors. By watching these diagrams one can observe that after 2 seconds and respectively 3 seconds, the reference signals have a sudden increase/decrease respectively decrease/increase in value, to test the system response to rapid change in the reference signal.

Compared with the results presented by S.E. Shafiei in [4], we achieved a better overall positioning error and a faster positioning when the signal abruptly changes. Besides the fact that our control system has better tracking capabilities, ours removes the overrides of the system along with a faster time in which the system reaches its goal position. Of course that the

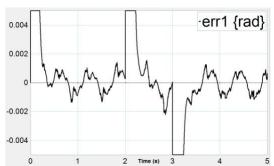
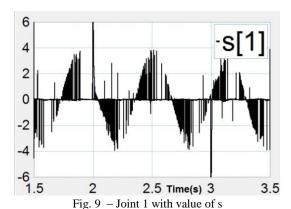


Fig. 7 – Joint 1 - Angular Error (rad)



To be able to analyze the sliding control method, we presented the values of s during the simulation. Figure 9 and 10 presents the value of s. Thus, one can observe how the sliding control method really works, by noticing how the computed value of the PID that returns the s value oscillates for different values of the reference signal. One can observe in figure 9 and 10 that when the two disturbances appear (at second 2 and 3), the value of s has really big amplitude peaks (over 30), due to its derivative part which has a punctual high value. As a difference between the two signals (joint 1 and joint 2) one can observe that in joint 2, the disturbance signal also records a big peak but it maintains a high value (over +/-10 rad) for another 0.125 seconds, where the signal of joint 1 presents only a peak

overrides eliminated are the ones that are very large, but the only ones that remain are those due to the system oscillation around its target, as one can see in the figures 7 and 8, which shows the magnified positioning errors of joint 1 and 2.

Figure 7 and figure 8 respectively, represent the angular error, which has been zoomed in to better observe the error variations. One can observe that in the case of joint number 1, the system reaches faster the reference signal, under 0.25 seconds, to an angular error lower than +/- 0.002 radians, meaning 0.11 degrees, after which the system starts to oscillate around the reference position due to the sinusoidal shape of the reference signal. In the case of joint number 2, the system reaches in the first 0.75 seconds from the disturbance (the sudden increase/decrease in reference value) a +/- 0.005 radians error meaning 0.28 degrees, after which the system starts to oscillate around the reference position due to the sinusoidal shape of the reference signal.



Fig. 8 – Joint 2 - Angular Error (rad)

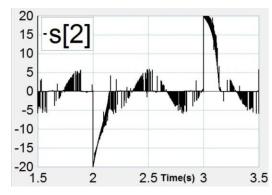


Fig. 10 – Joint 2 with value of s

at the moment of the disturbance and after a very short time (under 0.05 seconds) the signal lowers in value.

Because the Sliding Control is known for having a chattering problem we solved the issue by using a saturation function instead of a sign function and it can be seen that the output signal does not present this effect. The main observed conclusion is that this kind of control is one that tracks very well a uniform reference signal where other control methods like the simple PID controllers tend to overrides and in some cases to become instable if the PID gains are not correctly chosen. Also, the use of the fuzzy control method to adjust the output gain provides a better tracking error than the use of a PID control as shown by Vicente in 2003[10, 14].

In this paper, we designed the sliding mode control method through intelligent approaches that include sliding mode control, fuzzy control and bond graph modeling. This method uses the modeling approach of bong graphs used in designing and testing mechatronic systems but combined with the dynamic properties of the sliding mode control and fuzzy methods. Thereby, we showed a new method of robot walking control that improves the real time tracking control and we applied it on a 2 DOF mobile walking leg control. Compared to the conventional sliding mode control method, our system is 4 times faster and 10 times more accurate in tracking the position reference and can also compensate disturbances that may appear on the sliding surface. Comparing to other methods[4,13], which also uses a fuzzy based controller, our system has a better tracking error and much more, it can overcome uncertainties in a much shorter time with fewer error oscillations, resulting a robust controller with a predictive behavior.

The main performance demonstrates that using the sliding control method and a fuzzy adjustment law to calculate the gain matrix, removes the overrides and has better tracking capabilities, behaving like a predictive system, using the reference speed and acceleration and also the error speed so the robot can track and reach the reference position as smoothly as possible. By modeling the control system through Bond Graphs we could show more clearly the interaction between the components of the dynamic control system and also the electric motors, which can mimic the behavior of real life systems. Through simulation and modeling, and by using Bond Graphs, this paper develops a new research method that can analyze real time control and complex systems, specifically for the intelligent systems of which the mobile walking robots are part of.

The experimental results of the sliding mode control method which was used along with a fuzzy adjustment law in order to improve the performance of the robot dynamic control have proved that the studied robot system behaves very well when walking on an unstructured and bumped surface, because on encountering a sudden change in reference signal, the system is stable and compensates very well the disturbance.

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