How to Improve Positional Accuracy in Redundant Omnidirectional Mobile Robots?

Yaser Maddahi, S. M. Hosseini Monsef and Nikos E. Mastorakis

***Abstract*—** Non-systematic errors in wheeled mobile robots are significantly influenced by irregularities on the surface. The presence of non-smoothness on a surface causes the robot to deviate from its desired trajectory, and move towards an undesirable destination. This paper uses a technique, previously proposed by the first author, to alleviate the positional error originating from non-systematic resources during movement of a redundant omnidirectional wheeled mobile robot (OWMR). Kinematic equations of OWMRs form the foundation of this method to help us correct the robot motion, and reduce the errors occurring due to unwanted resources. To correct positioning errors, the expected surface on which the robot will be programmed to move is simulated. Afterward, a platform is fabricated having similar irregularities pattern. The robot is then programmed to travel on the designed platform and passes over designed obstacles. Two factors are obtained using experimental results: longitudinal and lateral. Both factors are then applied to the robot program. Then robot is finally tested on the same platform, and its motion accuracy is compared with the one obtained before applying the calibration factors. For studied case in this paper, non-systematic positional errors are reduced at least 80% that is a reasonable accuracy improvement.

***Keywords*—**Positional error; non-systematic error; wheeled mobile robot; omnidirectional wheel; kinematics.

I. Introduction

THERE exist two main types of error normally occurred during the mobile robot motion: systematic and non-systematic errors. Systematic ones originate from control and mechanical subsystems, which are caused by unavoidable imperfections during design, manufacturing and assembly processes. Non-systematic errors are caused by unexpected phenomena such as slippery floors, over acceleration, fast turning, external forces/torques and non-point wheel contact on the floor [1]. They are significantly influenced by irregularities on the surface such as bumps and cracks. Small obstacles cause the robot wheels to rotate more or less than desired rotation. Thus, the trajectory length travelled by robot will be changed. Since, it is almost impossible to predict or simulate the exact nature of surface irregularities to which the robot will be exposed, it is difficult to present a general quantitative test procedure for non-systematic errors [1]. Therefore, the non-systematic errors should decrease, or the robot should be calibrated, in order to achieve a desirable positioning error.

Calibration is defined as a set of operations that establishes, under specified conditions, the relationship between the values of quantities indicated by a measuring instrument and the corresponding values realized by standards [2]. The calibration approaches, used for calibrating mobile robots, include odometry [3], 3D camera error detection [4], active beacons [5], gyroscope [6], and magnetic compasses [7]. This paper focuses on odometry method. Odometry uses data from the movement of actuators to estimate change in position over time. As compared to other methods, odometry provides a better short-term accuracy allowing very high sampling rates at low costs [8, 9]. The purpose of odometry is to build an incremental model of the motion using measurements of the elementary wheel rotations [9]. For mobile robots, odometry remains to be one of the important means to achieve position error reduction. The odometry method can be applied to correct errors of all types of mobile robots including vehicle-type robots, robots with differential drive and omnidirectional robots.

With respect to Odometry, Tehrani *et al.* [10] developed a modified odometry system to increase positioning accuracy of a three-wheel mobile robot. They mounted the shaft encoders on three free-running wheels to avoid affecting the measurements of the sensors due to slippage of the driving wheels. Borenstein and Feng [11-13] introduced a method for measuring errors in differential drive wheeled mobile robots, and implemented it to correct errors for a number of robots including differential drive and omni-mate mobile robots. Maddahi *et al.* [3, 9, 14-16] applied the UMBmark benchmark test on different types of wheeled mobile robots. Both systematic [9, 14-16] and non-systematic [3] errors were corrected with this method confirming the significance and effectiveness of odometry method in the process of mobile robot calibration.

With respect to the use of odometry in calibrating mobile robots with omnidirectional wheels, Han *et al.* [17] focused on compensating errors of a four-wheeled OWMR which occur due to wheel slippage and bearing defects. Other sources of errors such as uncertainty in wheels diameters and differences in wheel diameters were not considered in their work. A new method has recently been proposed by the first author to reduce both systematic and non-systematic errors in OWMRs

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[18, 19]. The proposed method is used in this paper to calibrate the non-systematic errors of a four-wheeled omnidirectional wheeled mobile robots. The method was built using kinematic formulations of omnidirectional wheels, and was capable of compensating both systematic and non-systematic errors. Results showed that the method was very effective in improving position errors by at least 68%. In another work, they extended the method for a redundant WMR in which the Jacobian matrix is non-square and more complicated to implement [18].

The organization of this paper is as follows. Section II describes the prototype four-wheeled omnidirectional mobile robot followed and the corresponding kinematic formulation. Section III presents experimental results and performance evaluations. Conclusions are outlined in Section V.

II. MODELING OF ROBOT

A. Prototype robot

The prototyped robot has dimension of 8×8×9.5 cm^3 and weight of 375 g (Fig. 1). This robot has four omnidirectional driving wheels with single-row rollers and four motors. The diameter of the omnidirectional wheels is 70 mm and the width is about 10 mm. This robot is equipped with some infrared sensors to detect obstacles. Table I shows some key specifications of described mobile robot such as dimension, weight and maximum speed.

B. Kinematic modeling

The proposed technique is developed based on the kinematic formulations of omnidirectional wheeled mobile robot. The kinematic diagram of the prototype planar robot and associated wheel modeling are illustrated in Fig. 2. Each wheel is assumed to rotate independently. The coordinate systems $\{X_R O_R Y_R\}$ and $\{X_b O_b Y_b\}$ define the global (reference) and generalized (base) frames, respectively. To model this robot, the kinematic equations are firstly defined and then, based on these equations, non-holonomic constraints due to instant no-slip wheel conditions are written as follows [18]:

$$\begin{bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\\ \dot{\varphi}_3\\ \dot{\varphi}_4\end{bmatrix} = J\dot{\mu}_b \qquad (1)$$

where J is the Jacobian matrix, and is defined as follows [18]:

$$J = diag(\tfrac{1}{r_1\dot{\varphi}_1}\ \tfrac{1}{r_2\dot{\varphi}_2}\ \tfrac{1}{r_3\dot{\varphi}_3}\ \tfrac{1}{r_4\dot{\varphi}_4}). \qquad (2)$$

$$\begin{bmatrix} cos(\alpha_1+\beta_1+\gamma_1) & sin(\alpha_1+\beta_1+\gamma_1) & -l_1\,cos(\beta_1+\gamma_1)\\ cos(\alpha_2+\beta_2+\gamma_2) & sin(\alpha_2+\beta_2+\gamma_2) & -l_2\,cos(\beta_2+\gamma_2)\\ cos(\alpha_3+\beta_3+\gamma_3) & sin(\alpha_3+\beta_3+\gamma_3) & -l_3\,cos(\beta_3+\gamma_3)\\ cos(\alpha_4+\beta_4+\gamma_4) & sin(\alpha_4+\beta_4+\gamma_4) & -l_i\,cos(\beta_4+\gamma_4)\end{bmatrix} R(\theta_b)$$

TABLE I. SPECIFICATIONS OF FOUR-WHEELED PROTOTYPE ROBOT.

Variable	Value
Weight (kg)	0.375
Maximum speed of C. G. (m/min)	9.75
Wheel radius (cm)	3.0
Wheelbase (cm)	3.5
Encoder resolution (pulse/rev)	480
Dimension (L×W×H) (cm)	8×8×9.5

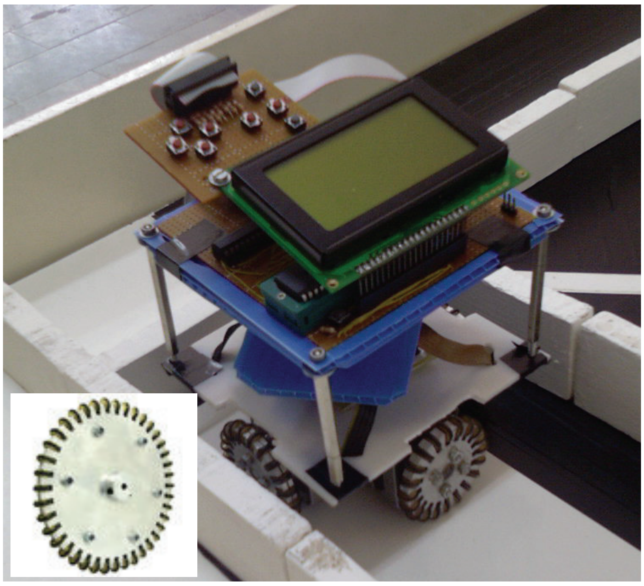

Figure 1. Prototype four-wheeled mobile robot. Inset shows a schematic of the omnidirectional wheel used.

In (1) and (2), $\mu_b = [x_b \quad y_b \quad \theta_b]^T$ is the robot posture with respect to the global coordinate. γ is the angle between the main wheel plane and the axis of rotation of the small circumferential rollers. β denotes the steering angle or angle of wheel plane relative to the robot main body which is usually constant. α is the angle between the wheel shaft and X_R axis when the robot is located in home position [19]. Moreover, $\dot{\varphi}$, r and l are the angular velocity vector, radius and wheelbase (the distance from the center of gravity of the robot to the center of wheels along a radial path) of the i^{th} wheel, respectively. $R(\theta_b)$ is defined as follows:

$$R(\theta_b) = \begin{bmatrix} cos\theta_b & sin\theta_b & 0\\ -sin\theta_b & cos\theta_b & 0\\ 0 & 0 & 1\end{bmatrix} \qquad (3)$$

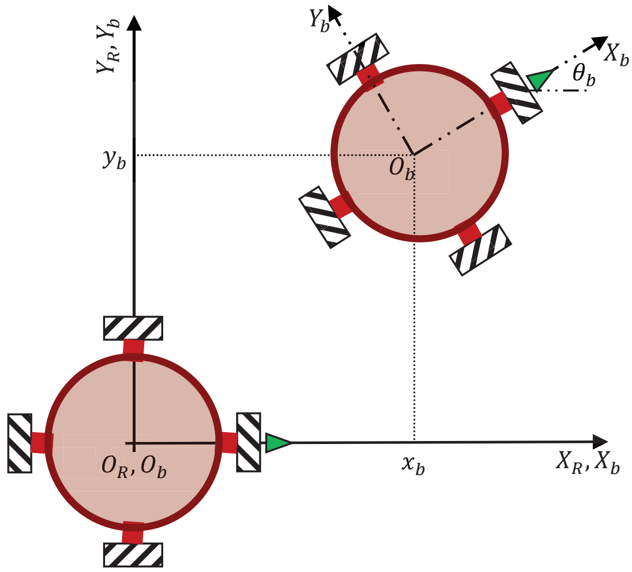

Figure 2. Coordinate systems of a prototyp four-wheeled mobile robot (revised from [18]).

III. Test Algorithm

Due to the significance of non-systematic errors effects on omnidirectional mobile robots movement, the compensation of non-systematic errors is investigated. Non-systematic errors are usually modeled by introducing surface irregularities using artificial obstacles which can be selected based on the geometry of real working environment of robot [1, 19]. Chosen obstacles for experiments are common electrical cable, such as the ones used in the following experiments, with 5 mm diameter, rounded shape and plastic coating. The distance between the wires should be chosen based on the condition of real environment as instructed in [19]. In this work, the cables are evenly placed along the trajectories (see Fig. 3). In experiments, used for validations in this study, the distance is chosen to be 30 mm in each straight leg with no obstacle located on the vertices. The process implemented here is in-line with previous study on calibration of non-systematic errors [1, 18] by other researchers. The robot is programmed to move along the straight path, shown in Fig. 3. Deviations of robot from the desired path are recorded in order to understand how much lateral and longitudinal positional errors appear when it completes the defined test. To organize the calibration procedure, two corrective coefficients are defined [19]. The first one is the lateral corrective matrix (F_{lat}), which is used to calculate the modified angular velocities in order to achieve the perfect movement along the straight trajectory [18].

$$F_{lat} = \begin{bmatrix} \frac{\dot{\varphi}_{1a}}{\dot{\varphi}_1} & & \cdots & 0 \\ & \frac{\dot{\varphi}_{2a}}{\dot{\varphi}_2} & & \\ \vdots & & \frac{\dot{\varphi}_{3a}}{\dot{\varphi}_3} & \vdots \\ 0 & & \cdots & \frac{\dot{\varphi}_{4a}}{\dot{\varphi}_4} \end{bmatrix} \quad (4)$$

The lateral corrective factor (F_{lat}) expresses the relationship between the wheel actual ($\dot{\varphi}_a$) and nominal velocities ($\dot{\varphi}$), by measuring the robot orientation angle (θ_e) and position errors (x_e and y_e). The orientation angle, θ_e, is measured using a protractor tool and the robot is commanded to move at a constant speed. Using (4), the lateral corrective factor appropriately to the robot motion equations such that the deviation angle, θ_e, converges to zero, *i.e.,* the robot maintains to stay along the desired path. However, even if the robot is aligned with the desired path, we need to further ensure that it reaches the desired location, *i.e.,* having no longitudinal error $\bar{x}_e$ [19]. This is done by equally adjusting the speeds of the wheels. Thus, a second coefficient, termed longitudinal corrective factor, F_{lon}, is defined [19]:

$$F_{lon} = \frac{L}{\sqrt{(L-\bar{x}_e)^2+(\bar{y}_e)^2}} \quad (5)$$

where L is the length of path and, $\bar{x}_e = \frac{1}{m}\sum_{i=0}^{m} x_{e,i}$ and $\bar{y}_e = \frac{1}{m}\sum_{i=0}^{m} y_{e,i}$.

As described in this section, the implemented compensates for the robot error using lateral (F_{lat}) and longitudinal (F_{lon}) corrective factors. The most integrated approach to implement these factors in the robot equations of motion, is to use them within the Jacobian matrix that relates robot trajectory (position and orientation) variables to the joints (wheels) variables, as shown in Equation (1). Considering the defined corrective factors, the final angular velocities, needed to correct both lateral and longitudinal errors, are defined as [19]:

$$\begin{bmatrix} \dot{\varphi}_{1f} \\ \dot{\varphi}_{2f} \\ \dot{\varphi}_{3f} \\ \dot{\varphi}_{4f} \end{bmatrix} = F_{lon} F_{lat}\, J \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_e \end{bmatrix} \quad (6)$$

where F_{lat} and F_{lon} are obtained using (4) and (5), respectively. J is defined by Equation (3). As observed, for prototyped robot, β and γ are zero. $\dot{\varphi}_f$ indicated the final angular velocity of each wheel which should be entered in the robot program.

IV. Experimental Results

A. Calibration indices

Two metric proposed in [18] and [19] are used to investigated the effectiveness of the proposed method:

- Radial error (δr_e) which is defined as follows:

$$\delta r_e = \frac{1}{m}\sum_{i=1}^{m}\sqrt{\left(x_{e,i}\right)^2 + \left(y_{e,i}\right)^2} \quad (7)$$

$x_{e,i}$ and $y_{e,i}$ are the longitudinal and lateral positional errors, and are defined in Fig. 3.

- Mean error improvement index (δr_m) [19]:

$$\delta r_m = \frac{(\delta r_{m,BF} - \delta r_{m,AF})}{\delta r_{m,BF}} \times 100\% \quad (8)$$

In (8), $\delta r_{m,BF}$ and $\delta r_{m,AF}$ are the mean values of radial errors before and after calibration, respectively.

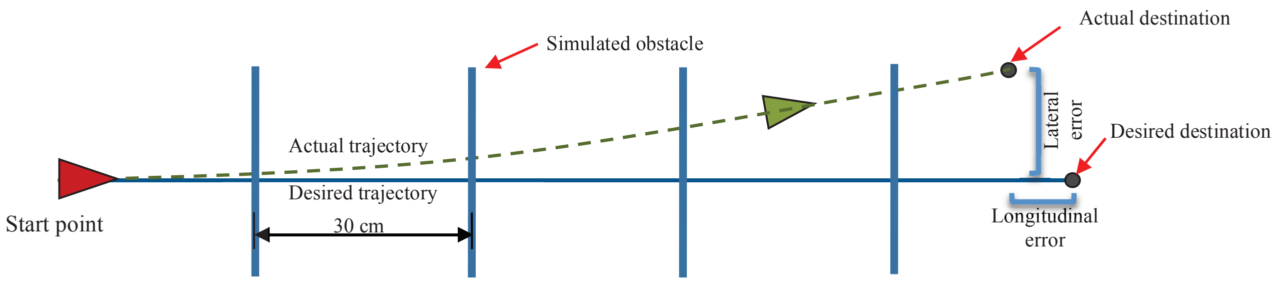

Figure 3. Test trajectories used to coorect non-systematic errors. Vertical lines show simulated obstacles (revised from [21]). Robot is expected to move along the desired trajectory (solid path); while it goes toward an undesirable (actual) destinationdue to the presence of floor irregularities. The positional difference between the actual and desired destinations is alleviated using this odometry-based technique.

B. Positional errors compensation

Based on the described test technique for this robot, the equations of robot were applied to the prototype robot and over ten trial runs. Figures 4a and 4b illustrate the position errors of the four-wheeled OWMR before and after calibration. In this test, the robot was programmed to travel along two perpendicular trajectories, when (a) the first ($\alpha_1 = \pi/2$) and third ($\alpha_3 = 3\pi/2$) wheels move and the other wheels have no angular velocity (path 4W-1), and (b) the actuators of wheels 1 and 3 are turned off and only the second ($\alpha_2 = \pi$) and fourth ($\alpha_4 = 0$) wheels rotate with the same and opposite angular velocities (path 4W-2). The paths used in this test are depicted in Fig. 4. As observed, the robot motion was corrected with proposed technique, *i.e.,* positional errors converged to zero center of coordinate system in both robots. In the case that more accuracy is needed, the test should be repeated in order to obtain the secondary corrective factors and to re-modify the motion. The new factors will be applied to robot program, Thus, the robot will be influenced by two sets of corrective factors to achieve reasonable position error.

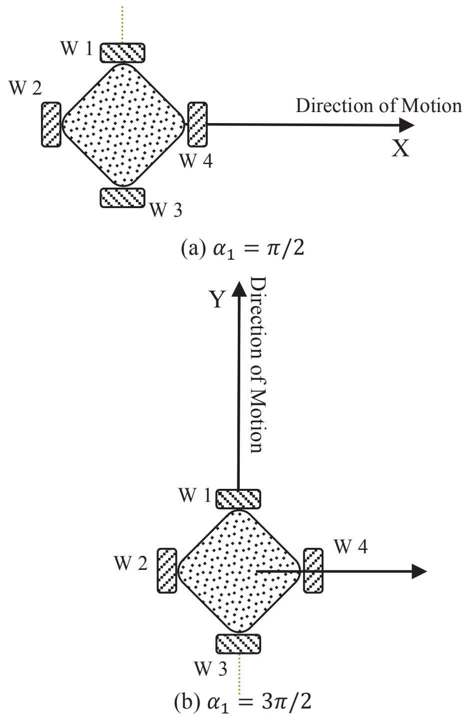

Figure 4. Defined path directions on which the rbot was programmed to move: (a) 4W-1, (b) 4W-1 (recised from [18]).

Table II illustrates the amount of average errors for both robots before and after calibration as well as estimates of Skewness, Kurtosis from measured data. Also, the lateral corrective matrix and longitudinal corrective factor are shown in this table. The sixth column of Table II presents the percentage of error improvement (δr_m) to the variability of data over ten trials, standard deviation is used to measure confidence in statistical conclusions. The results obtained from experimental tests showed that the proposed method is capable of calibrating the errors in prototyped mobile robot. As shown, for tested robot, the mean value of mean values of radial error are improved up to 80%, respectively.

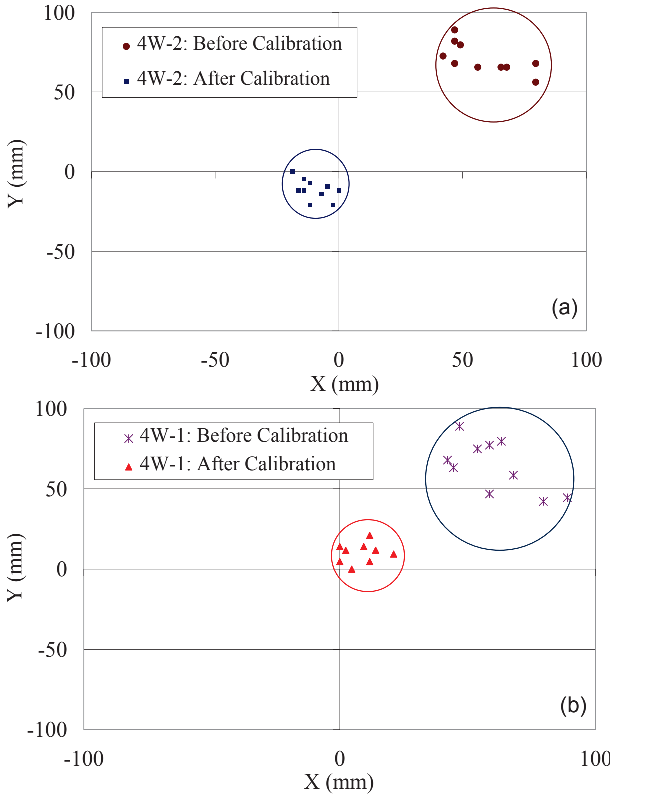

Figure 5. Radial non-systematic positional error of 4W robot in 4W-2 (Fig. 4a) and 4W-1 (Fig. 4b) paths.

TABLE II. TEST INDICES FOR NON-SYSTEMATIC TESTS BEFORE (BF) AND AFTER (AF) CALIBRATION.

	$F_{lat,n}$	$F_{lon,n}$	Mean Error	δr_m (%)
BF	$\begin{bmatrix}1.08 & & & \\ & 0.96 & & \\ & & 1.02 & \\ & & & 0.98\end{bmatrix}$	0.94	91.72	80
AF			15.43	

V. CONCLUSIONS

A kinematic-based calibration technique was reported for accurate calibration and error reduction of a four-wheeled omnidirectional mobile robot. We, firstly, provided an overview of mobile robots positioning methods and then, presented a technique capable of calibrating omnidirectional mobile robot with various mechanisms to correct non-systematic errors. Next, the test method was used to correct the errors of a prototyped omnidirectional mobile robot with omnidirectional wheels. It was demonstrated that the proposed technique is simple to implement and leads to good and reasonable error improvement percentage. Specifically, experimental results showed that the non-systematic errors were improved at least 80%. Based on experimental results done for the prototype robot, the method is helpful as a potential method for calibration of non-systematic errors in robots with omnidirectional wheels. With reference to work done in [18] and [19] and results presented in this paper, this method was found as a helpful tool to reduce non-systematic positional errors.

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