Analysis of Noise Level Exceedances by Exponential Rate Function in Non-Homogeneous Poisson Model

Claudio Guarnaccia, Joseph Quartieri, Nikos E. Mastorakis and Carmine Tepedino

***Abstract*—** The study of the impact of acoustical noise on human activities is a very important issue in cities and areas in which relevant noise sources are present. The effect on health, both auditory and non-auditory is largely documented in literature and several models have been developed to take care of this problem. Since almost all the national regulations fix a maximum acoustical level, according to the area and to the kind of buildings and activities that occur in that, a model based on threshold exceedances study is suitable. In this paper, a non-homogeneous Poisson model is presented and applied to a large dataset of noise measurements. The parameters probability distributions estimation, based on Monte Carlo Markov Chains and Gibbs algorithm, will be described. The posterior distributions of the parameters will be shown and their mean values will be used to plot the cumulative mean function. This function, that represents the number of surpassings of the threshold as a function of the time, can be compared with the observed exceedances.

***Keywords*—** Acoustical Noise Level, Cumulative Rate Function, Exponential Rate Function, Goel Okumoto Function, Non homogeneous Poisson Process, Predictive Model, Threshold Surpassing.

I. INTRODUCTION

ACOUSTICAL noise pollution is one of the major problems in urban areas [1]. Together with air pollution, in fact, it is a relevant risk for human health. In [2], [3] some of the effects related to the exposure to acoustical noise are resumed, focusing on the auditory and non-auditory effects.

The most important source of noise are transportation infrastructures and industrial areas, such as documented in [1], [4], [5]. Many models, with different approaches, have been developed to predict the noise levels produced by trains (for instance [6]-[11]), road traffic (for instance [12]-[23]), airport (for instance [24], [25]), industrial settlements (for instance [3], [26]), etc..

In this paper, the approach that will be pursued is based on the implementation of a non-homogeneous Poisson process [27], [28] to study the probability of surpassing of a certain noise threshold. Similar models have been presented in [29], [30] for air pollution and [25], [31] for noise level. The introduction of changepoints is discussed in [25] for airport noise, while in [32] the comparison between various models, with a different number of changepoints, is discussed on four large dataset of noise levels.

In this paper, one of these datasets will be adopted to test the implementation of a different rate function, that is exponential instead of power law. In particular, the Goel Okumoto (GOP) rate function [33] will be implemented instead of the Weibull Power Law (PLP) rate function, adopted in [25] and [31].

The dataset is related to acoustical noise equivalent level measured in Messina, Italy, in the framework of the long term monitoring campaign promoted by the Department of Urban Mobility of the city. The levels considered here are measured during the night and are evaluated on a 8 hours range, from 10 p.m. to 6 a.m..

Since some data were missing, both single and group of data, a Time Series Analysis model, such as the ones developed in [34]-[37], has been used to fill the gaps. This will be the same procedure adopted in [32] to make the datasets continuous.

The results will be presented in terms of comparison between observed and estimated exceedances, i.e. cumulative mean function.

In addition, the evaluation of the probability of observing a certain number of threshold surpassings will be presented in the last section, showing how the model can be helpful in decision processes by policy makers.

II. MODEL PRESENTATION

The Non-Homogeneous Poisson Process (NHPP) is a stochastic model able to count occurrences of a certain event, in our case the surpassing of a chosen noise level threshold.

Let $\mathrm{N}^{(\theta)} = \left\{\mathrm{N}_t^{(\theta)} : t \in [0, \mathrm{T}]\right\}$ be a NHPP with mean value function $\mathrm{m}(\mathrm{t}|\theta)$ and θ the parameters vector. The function $\mathrm{m}(\mathrm{t}|\theta)$ represents the cumulative function of the expected number of events registered by $\mathrm{N}^{(\theta)}$ up to time *t*. In this paper, the events are represented by the surpassing of a certain acoustical noise threshold (68 dBA). In order to fully characterize the process, the functional form of $\mathrm{m}(\mathrm{t}|\theta)$, or

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equivalently, of its rate function $\lambda(t|\theta)$, must be achieved. In fact, the relation between the two functions is:

$$\lambda(t|\theta) = \frac{d}{dt} m(t|\theta) \quad . \tag{1}$$

The probability to observe k exceedances in the time range [*t; t+s*] is given by:

$$P(N_{t+s}\text{-}N_t\text{=}k) = \frac{[m(t+s)-m(t)]^k}{k!}\, e^{-[m(t+s)-m(t)]} \tag{2}$$

where Nt is the number of times a threshold is surpassed in the time range from 0 to *t*.

The model adopted in this paper is based on the Goel-Okumoto (GOP) [33] process defined by the following mean value function:

$$m(t|\theta) = \alpha[1-\exp(-\beta t)], \quad \alpha,\beta > 0 \ , \tag{3}$$

where $\theta = (\alpha, \beta)$. The rate function associated with this process is given by

$$\lambda(t|\theta) = \alpha\beta \exp(-\beta t). \tag{4}$$

In the Messina dataset considered in this paper, the epochs of occurrence of noise level threshold violations up to time *T* are included in the dataset $D_T = \{n;\ t_1, \dots, t_n\ ; T\}$, in which *n* is the number of observed threshold overruns times which are such that $0 < t_1 < t_2 < \dots < t_n < T$.

The likelihood function for θ is:

$$L(\theta|D_T) = \left(\prod_{i=1}^{n} \lambda(t_i|\theta)\right) \exp\big(-m(T|\theta)\big) \tag{5}$$

A. *Parameters estimation*

In this work, in order to have the estimation of the model parameters, an empirical Bayesian analysis is applied. The choice was to use approximately non-informative prior distributions, in particular uniform U[*a,b*] distributions, with *a* and *b* chosen in an appropriate way.

The software OpenBugs has been used to obtain simulated samples from the posterior distribution of θ. In this software framework, only the distribution for the data and prior distributions for the parameters need to be specified. This software adopts standard Markov chain Monte Carlo (MCMC) methods. For further details, one may refer to [25], [29]-[32].

III. ANALYSIS AND RESULTS

The model presented above has been tested on a dataset of 1341 noise measurements, collected in Messina, Italy, in a monitoring station placed in Boccetta street, during night time (from 10 p.m. to 6 a.m.). The time range goes from the 11th of May 2007 to the 10th of January 2011. The observed exceedances of the threshold (68 dBA) are 900. The statistics of the data are resumed in Tab. 1.

Table 1: Summary of the statistics of the 1341 noise data

	Mean	Std.dev	Median	Min	Max
1341 data	68.25	1.31	68.50	63.5	72.0

Five chains have been chosen in the two runs, with 100000 iterations each and different starting points. The parameters have been evaluated after a burning period of 50000 iterations and thinning every 10 steps. The choice of the prior distributions is crucial in the Bayesian approach. In this paper, a uniform distribution U[*a,b*] has been chosen both for α and β, with opportune boundaries *a* and *b*. An initial strongly informative distribution choice, in general, should be avoided but, once the first iteration has been run, more informative distribution can be used.

The uniform prior distributions, with *a* and *b* parameters (boundaries), are:

$$\alpha \rightarrow U[1000\ , 3000]$$

$$\beta \rightarrow U[0.0001\ , 0.00055].$$

The starting points of the five chains have been choosing in all the interval [*a, b*] and are:

1. $\alpha = 1100$; $\beta = 0.0005$;
2. $\alpha = 1800$; $\beta = 0.00045$;
3. $\alpha = 2500$; $\beta = 0.00012$;
4. $\alpha = 2100$; $\beta = 0.00035$;
5. $\alpha = 2900$; $\beta = 0.0003$.

The convergence of the five chains is achieved quite soon, independently from the starting points resumed above, as shown in Fig. 1 and Fig. 2.

The resulting posterior probability density functions of the parameters are reported in Figg. 3 and 4, while the statistics are resumed in Tab. 2.

With the mean values of the parameters, the mean function can be plotted versus time and compared with the observed surpassings of the threshold. This comparison is reported in Fig. 5.

As it can be noticed, the agreement between the two curves is very good, except for a region that goes approximately from 800 to 1150. In this time interval, a lower number of exceedances is observed but the model is not able to follow this pattern. A possible solution is represented by the introduction of a suitable number of change-points, such as in [25], [30], in order to evaluate the parameters in different ranges and with a better agreement. This improvement of the model is part of an ongoing work [32].

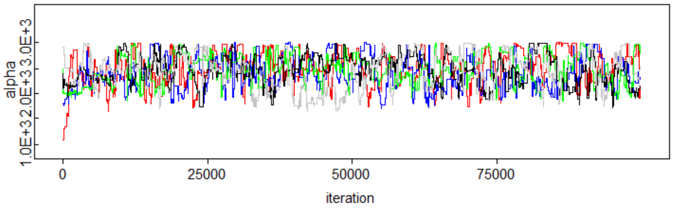

Fig. 1: Time history of the 100000 samples generated in the MCMC procedure used to estimate alpha parameter

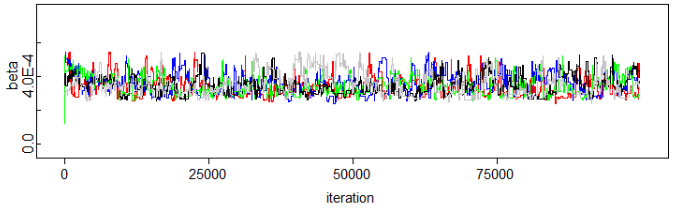

Fig. 2: Time history of the 100000 samples generated in the MCMC procedure used to estimate beta parameter

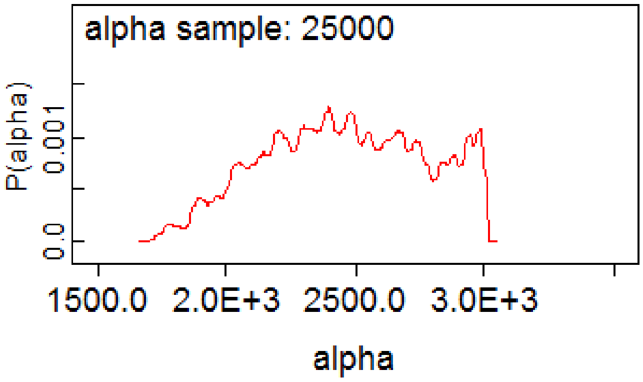

Fig. 3: Density plot of the alpha posterior distribution

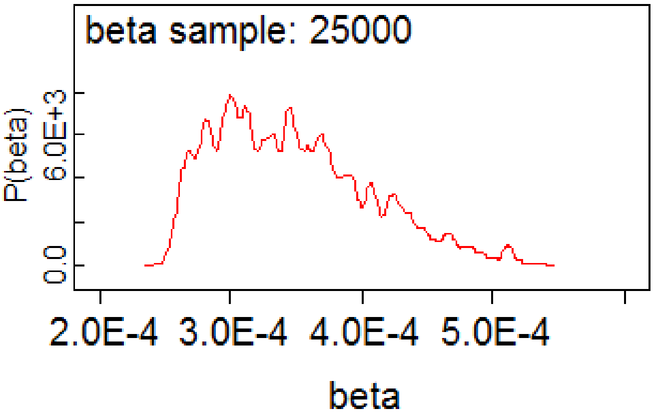

Fig. 4: Density plot of the beta posterior distribution

Table 2: Summary of statistics of the α and β parameters

	Mean	Std.dev	Median	val 2.5%	val 97.5%
α	2462.0	310.0	2460.0	1884.0	2989.0
β	$3.5\ 10^{-4}$	$5.9\ 10^{-5}$	$3.4\ 10^{-4}$	$2.6\ 10^{-4}$	$4.8\ 10^{-4}$

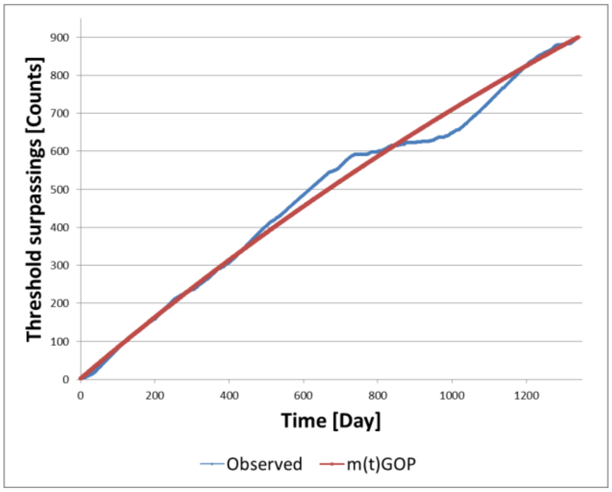

Fig. 5: Observed (blue line) and predicted by the GOP model (red line) threshold surpassings plotted versus time

IV. PROBABILITY OF THRESHOLD SURPASSINGS

In this section, the authors will present how the Poisson distribution can be used to evaluate the probability of observing a certain number *k* of events of noise level threshold surpassings, in an interval of *s* periods, starting from the period *t*. Let us remind that the time base for the equivalent level is 8 hours, from 10 p.m. to 6 p.m. (night level), thus each period corresponds to a night measurement.

In order to define a suitable Poisson distribution, the mean values of the parameters of the GOP model rate function are needed. These parameters have been evaluated in the previous section and their statistics are reported in Table 2.

The simulation reported here is based on an interval of 30 periods (*s*), i.e. 30 days, starting from period 1200 (*t*), i.e. the 22nd of August 2010, in the case of 68 dBA noise level threshold. The actual observed exceedances are 23.

Figg. 6 and 7 report respectively the probability to observe *k* threshold exceedances and the cumulative probability of observing up to *k* threshold exceedances in the 30 periods time interval.

Table 4, instead, presents the following data:

1) probability of observing exactly the real number of threshold surpassings;
2) cumulative probability of observing up to 10 threshold surpassings;
3) cumulative probability of observing up to 20 threshold surpassings.

It can be noticed that the probability of observing the real number of exceedances is quite low (about 3%), that means that looking for the exact value of threshold surpassings is not the most useful way to use this tool.

On the contrary, the other two results are more relevant. The cumulative probability (*p*) of observing up to 10 surpassings is very low, i.e. there is a high probability (1-*p* is about 95%) of having more than 10 exceedances in the 30 periods (days) that follow the 22nd of August 2010 ($t = 1200$).

Finally, the cumulative probability of observing up to 20 surpassings is about 81%, stating that there is an almost low probability (1-p is about 19%) of observing more than 20 surpassings of the threshold. These results show that the model is able to give strong indications about the range of the threshold exceedances. In the case presented here, the indication is that the site is affected by many threshold surpassings and it is critical from the noise pollution point of view.

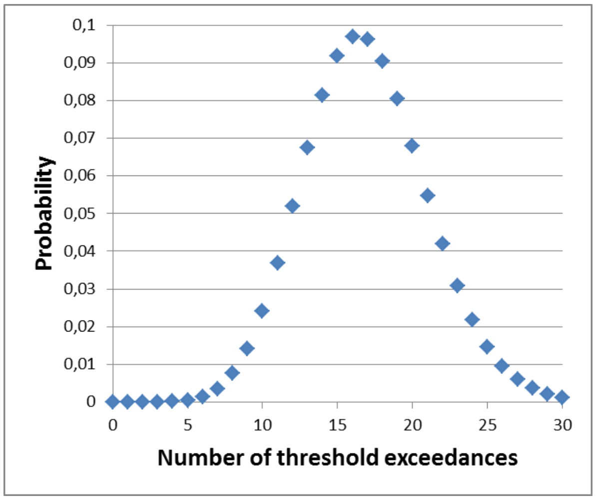

Fig. 6: Probability to observe k exceedances in an interval of 30 days from the period 1200, as a function of k

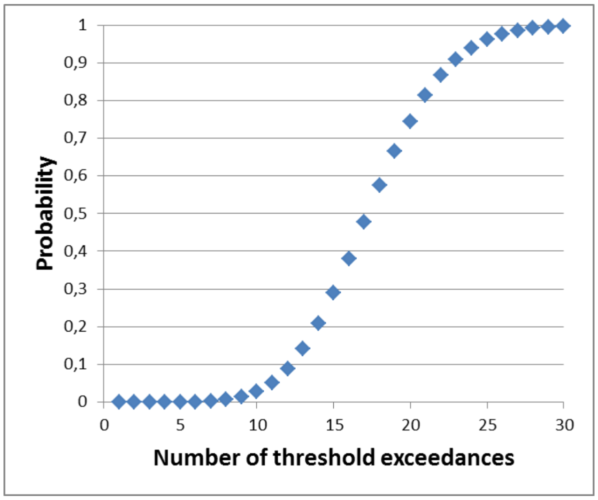

Fig. 7: Cumulative distribution function of observing up to k exceedances in an interval of 30 days from the period 1200

Table 3: Probability of observing the actual number of exceedances (k=23) and cumulative probabilities of observing up to 10 and up to 20 threshold surpassings

Probability $P(k = 23)$	Probability $p(k \leq 10)$	Probability $p(k \leq 20)$
0.030815	0.051557	0.812575

V. CONCLUSIONS

In this paper, the problem of predicting the probability of surpassing of a certain noise level threshold, on a given time range, has been considered.

The problem has been attached by means of non-homogeneous Poisson process and the key point of the presented work has been the choice of the rate function form. An exponential rate function has been introduced, according to Goel Okumoto (GOP), instead of the Weibull Power Law (PLP) rate function, adopted in previous papers.

The model parameters estimation has been performed in the OpenBugs software framework, adopting a MonteCarlo Markov Chain technique. The uniform prior distribution choice for parameters probability density and the chains starting points resulted in an almost quick convergence of the chains. The posterior distributions have been presented and the mean values used to calculate and plot the mean function, i.e. the cumulative function of the surpassings predicted versus time.

The comparison of the model results with the real observed exceedances is good, except for a region of the plot in which a lower number of surpassings is measured, with respect to the prediction of the model.

The evaluation of the probability of observing a certain number of threshold surpassings has been presented in the last section, showing that the model is able to give strong indications about the range of threshold exceedances.

The implementation of more informative prior distributions and of different rate functions in the Poisson process represent the future steps of this work, in order to achieve an always better prediction of the probability of noise level threshold surpassings.

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