Ambient vibration testing and operational modal analysis of a historic tower

M. Diaferio, D. Foti, and N. I. Giannoccaro

***Abstract*—**An accurate knowledge of the dynamical parameters of structures is definitely useful for seismic assessment and for the design of risk mitigation interventions. In this paper, the opportunities provided by dynamic identification techniques for the non-destructive evaluation of heritage structures are discussed with focus on the bell tower of Announziata (Corfù, Greece), a masonry tower, which shows a high damaged scenario and, consequently, a high vulnerability to dynamic and seismic forces. The paper presents the experimental investigations and operational modal analysis results, useful for defining the finite element model of the tower. The monitoring system consists of several elements properly connected: in total twenty-four accelerometers have been positioned, eight for each of the three floors and oriented according to the orthogonal directions x and y. This configuration has been also conditioned by many operative problems about the position of the instrumentation due to the limited accessibility of the structure, not only to the main access but also to reach the top. It is important to emphasize that the data obtained are not connected to external events detected during the acquisitions, so it is possible to identify with a certain confidence the first six frequencies of the tower and their corresponding mode shapes.

***Keywords*—** Non-destructive tests, operational modal analysis, model updating, dynamic analysis.

I. Introduction

IN the preservation of architectural heritages it is important a careful study of the structure and of its dynamic characteristics in order to describe its actual behavior [1-3]. The necessity of identifying unknown geometrical data and material properties is due to the usual impossibility of conducting classical tests for their evaluation; so the numerical models may be validated only by means of non-destructive techniques. In literature various methods have been proposed for estimating the mechanical properties of the structural materials also in the presence of retrofitting interventions by means of innovative materials [4-9], however these procedures allow to evaluate only local properties and the extension to the global structure is too much burdensome for practical applications. Other approaches makes use of the experimental evaluation of the modal parameters of the structure. The modal parameters may be compared with those of the model and the unknown materials and geometrical parameters may be estimated for obtaining an accurate FE model. These techniques have the advantage of providing information on the overall structure with a reasonable experimental effort [10-32].

In this paper, the case of study is the bell tower of Announziata, a masonry tower (Fig. 1), which shows a high damaged scenario. The bell tower of the church is located in the town of Corfù, Greece. The church was built in 1394 and it was one part of a Roman monastery, nowadays the only existing one. The interior of the church [1] contained the tombs of generals who died in the sea battle of Naupactus (1571). The church of Announziata was bombed during the second world war and it suffered from cracking; then it collapsed in March 1952 when long term phenomena and earthquakes contributed to increase the damage. So, from 1952 only the campanile of the medieval monastery has survived to the present day with heavy cracking. It is possible to observe deep vertical and diagonal cracking, deterioration of mortar on the stone walls and development of creepers in the masonry. The bell tower is a stone tower with an almost square plan section. A double arch (Fig. 1) supported by a stone column in the center is present at each of the four façades.

Fig. 1. Announziata bell tower.

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The structure of the tower is made with masonry walls and the three floor systems made by solid bricks are supported by vaults. At the top of the tower there is the bell cell with four bells. The current state of the tower may be defined critical, with many damages and cracking.

The possibility of defining a detailed Finite Element (FE) model of the tower taking into account the experimental modal identification data may be considered very important for getting information about the structural health of the tower. To this aim, the present paper shows all the details and results of the experimental modal identification procedure able to complete the preliminary analysis presented in [31-32].

II. Experimental setup and identification procedure

The Announziata tower was instrumented with 24 high sensitivity seismic accelerometers ICP PCB 393B31 placed on 12 positions (labelled with numbers 1-12 on the plans in Fig. 2) on 3 different levels; 8 accelerometers were placed on the four corners of the first floor (position 1-4 in Fig. 2b), 8 were placed on the four corners of the second floor (number 5-8 in Fig. 2b), and 8 were placed on the basis of the four columns (numbers 9-12 in Fig. 2b). Appropriate rectangular blocks were designed and realized in order to ensure the orthogonality of the couple of accelerometers placed on the same position. The accelerometers were inserted with screws on the threads realized on the perpendicular faces of the block).

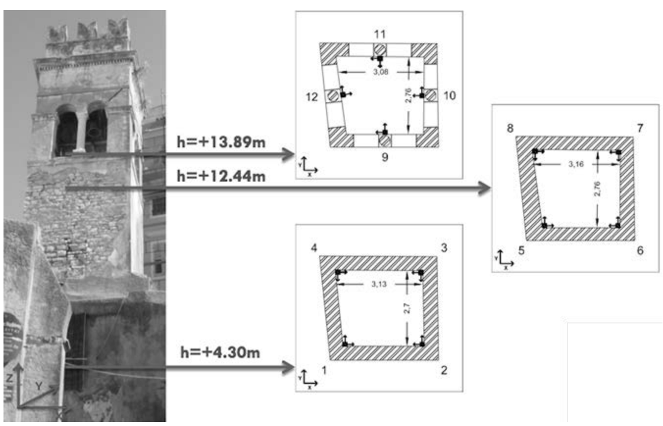

Fig. 2. Annunziata bell tower and layout of the experimental setup.

It was not possible to achieve the belfry of the campanile to place accelerometers on the top of the tower; also it must be considered the very compromised situation of the building, let fall into decay in the last years. It was very difficult, also, to reach the first floor.

The bell tower is positioned in the city center, in a crowded area especially during the day time. There is a main entrance road open to the car passage and a rotatory (Fig. 3) very close to the tower where cars and trucks can select the preferential way. The continuous passage of bikes, cars, motorcars and pedestrians has characterized all the environmental tests.

Preliminary tests were carried out on 12th October 2012; in the day after several consecutive tests were conducted. The data acquisition was carried out by recordings of 10 minutes with a frequency of 1024 Hz, which has been subsequently decimated by a factor equal to 4 to have a frequency of 256 Hz. About 10 consecutive acquisitions were carried out and, in each acquisition, all the relevant events (passage of cars, motorbike, other possible disturbs), were noted. An heavy rain, unfortunately, occurred in the night between 12th and 13th October, and infiltration of water was verified on the instrumentation that was placed on the second floor and on the basis of the columns.

Fig. 3. Position of the Announziata bell tower.

III. Identification results

The analysis of the experimental results was subsequently performed. A specific software [33] was used for the extraction of the modal parameters from ambient vibration data. The model defined by means of this software is shown in Figure 4 with the corresponding reference system *xyz*.

A preliminary analysis was conducted on the time series recorded by the accelerometers for evaluating the effects of the urban traffic and the functionality of the accelerometers considering the difficult environmental conditions.

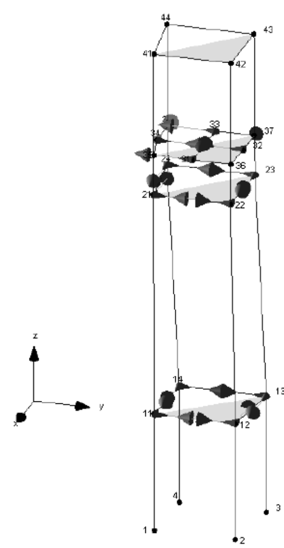

Fig. 4 Reference system and model for the identification

The preliminary analysis allowed to individuate that some accelerometers were not performing properly: the ones placed in positions 23 (both directions *x* and *y*), 24 (direction *x*) and 32 (direction *y*) (see the model in Fig. 4). In addition, the preliminary analysis permitted to clearly highlight the effects of external disturbances. In Figs 5-7 the time histories of three opportunely selected tests (named a, b, c) are shown in relation to the accelerometers placed in positions 21 and 22 of Fig. 4 (both the directions *x* and *y*). It is evident from Figs. 5-7 the extreme sensibility of the experimental setup to external events: for test a (Fig. 5), there was heavy and continuous traffic after 260-280 seconds and some spurious events after about 440, 540 and 560 seconds from the starting time. For test b (Fig. 6) there are not evident effects on the time histories, while on test c (Fig. 7) heavy traffic effects are evident at the beginning of the registration (80-120 sec) and at the end of the registration (400-480 sec) for the accelerometers along *x* axis (left column of Fig. 7), while some spurious events are evident after around 510 sec. The time histories of the selected tests seem to be very different each other for the external disturbances that influence the accelerometers data.

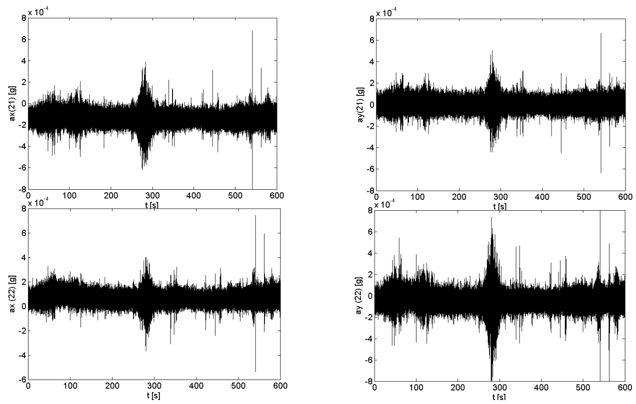

Fig.5 acquisition signals for points 21 and 22 for test a.

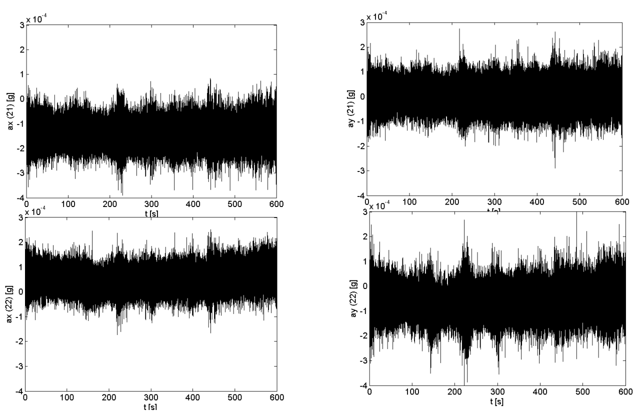

Fig.6 acquisition signals for points 21 and 22 for test b.

It is important to emphasize [32] that the data results are closely connected to external events detected during the acquisitions, so the preliminary analysis was conducted on the time signals of the accelerometers. For this reason, a wide experimental analysis was carried out repeating the acquisition for 14 times in two consecutive days.

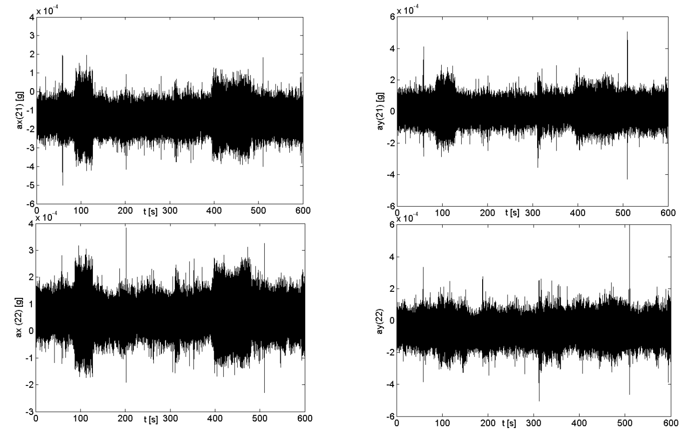

Fig.7 acquisition signals for points 21 and 22 for test c.

The frequencies of the building and the modal shapes time histories were identified for all the tests with a simple operational modal analysis (OMA). In particular, for each analysis [33] two different OMA methods were used: the Enhanced Frequency Domain Decomposition (EFDD) in the frequency domain and the Stochastic Subspace Identification (SSI) using Unweigthed Principal Components (UPC) in the time domain [34-35]. The estimated frequencies for all the tests are named using the number of the date (first number 1 or 2 considering the first or second day) and the number of acquisition (second number 1, 2…5 for the first day, 1, 2…9 for the second day). Table 1 shows the frequencies evaluated by means of the SSI method. The identified frequencies are consistent for all the tests and for both methods; the SSI method permits to evaluate in each case all the frequencies, while the EFDD method in some cases is not able to identify the frequencies of higher order.

Table 1. Identified frequencies [Hz] with SSI method for all the tests

Fr.	1-1	1-2	1-3	1-4	1-5	2-1	2-2	2-3	2-4	2-5	2-6	2-7	2-8	2-9
1	2.61	2.61	2.61	2.60	2.61	2.63	2.62	2.62	2.63	2.62	2.62	2.63	2.65	2.63
2	2.83	2.83	2.82	2.81	2.82	2.82	2.82	2.84	2.83	2.82	2.82	2.84	2.84	2.83
3	5.50	5.47	5.48	-	5.46	5.52	5.44	5.51	5.51	5.50	5.51	5.52	5.55	5.55
4	7.04	7.05	7.05	7.05	7.05	7.05	7.04	7.02	7.03	7.07	7.03	7.05	7.07	7.06
5	8.04	8.01	8.03	7.99	8.021	8.01	8.01	8.01	8.04	8.05	8.03	8.05	8.0	8.03
6	11.28	11.3	11.27	11.14	11.19	11.32	11.25	11.28	11.3	11.37	11.31	11.29	11.32	11.29

In Table 2 a statistical analysis of the identification results is shown: it can be observed that the mean values are almost the same for both methods and that the standard deviation is very low ensuring a good repeatability and consistence of the identified frequencies nevertheless the different casual events. The frequencies identified by means of the mean value may be considered a stable experimental benchmark that characterizes the dynamical behavior of the tower.

Figs. 8 and 9 show the identification diagrams using SSI and EFDD methods, respectively, for test 2-4.

The identified frequencies were analyzed for detecting the mode shapes. It was observed that in all the tests of Table 1, the six identified mode shapes perfectly correspond to the ones expected for the tower. In detail, the first and second frequency were identified as the first couple of flexional modes on the *y* and *x* axes, respectively, the third frequency was the torsional mode, the fourth and fifth were the second couple of flexional modes on the *y* and *x* axes, respectively, and the sixth is the second torsional mode. Fig. 10 shows graphically the experimental identified mode shapes for test 2-4 by mean of the SSI method.

Table 2. Statistical analysis about the identified frequencies considering all the 14 performed tests

Frequency number	Mean value [Hz] (SSI)	Standard deviation (SSI)	Mean value [Hz] (EFDD)	Standard deviation (EFDD)
1	2.625	0.0125	2.616	0.0157
2	2.832	0.0078	2.828	0.0071
3	5.505	0.0325	5.526	0.0220
4	7.052	0.0152	7.067	0.0526
5	8.027	0.0191	8.051	0.0497
6	11.280	0.0571	11.323	0.1137

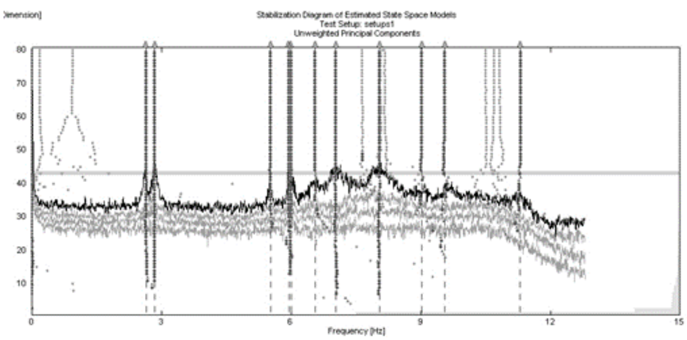

Fig.8 Identification by using SSI for test 2-4.

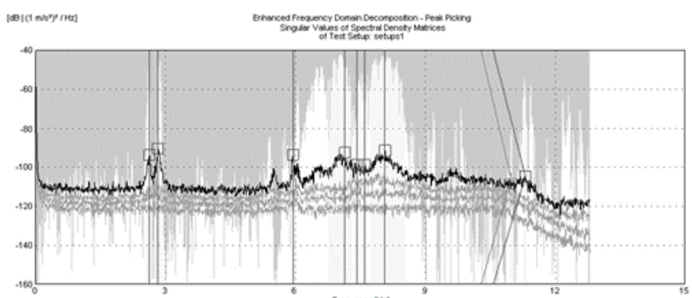

Fig.9 Identification by using EFDD for test 2-4.

IV. FINITE ELEMENT MODEL

A preliminary finite element (FE) model of the Announziata tower has been realized considering the plan section as perfectly squared. The structure is realized with brickwork walls and the three floor systems are made by full bricks supported by vaults that in turn are supported by the walls parallel to the N-S axis of the building.

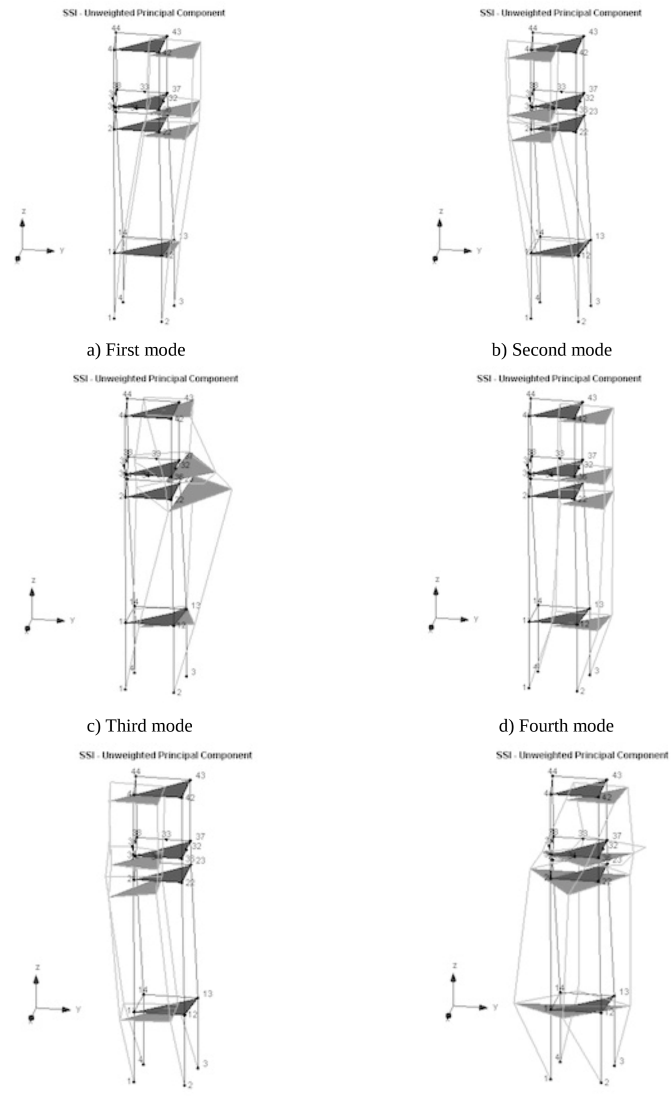

Fig.10 Identification of mode shapes of test 2-4 using the SSI method.

The materials characteristics (characteristic strength in compression f_{wd}, elastic modulus E, weight per unit volume γ_s and mass per unit of volume m_s) have been deduced by [1] where the properties have been defined on the basis of Eurocode-6. In [1] the pattern of vertical cracking has been taken into account considering a reduction of the elastic modulus of the masonry; in detail, it has been adopted the following elastic modulus for the cracked masonry:

$$E = \frac{2}{3} \cdot E_{initial} \tag{1}$$

where $E_{initial}$ is the elastic modulus of the undamaged masonry. The considered material properties for the vertical walls of the tower (material 1), for the three solid brick floors of the campanile (material 2), and for the stone column (material 3) are reported in Table 3. Finally for the vaults filling material γ_s has been assumed equal to 10 kN/m^3.

The three dimensional FE model of the tower has been defined by means of the SAP2000 [36]. The model has two typologies of elements, the ‘frame’ and the ‘shell’. The frame ones are prismatic linear elements used to model structural components such as the stone columns supporting the bell tower openings and the bells’ supporting framework (Fig. 11a). The shell elements have been used for modeling the masonry walls, such as the vertical walls; for the vaults caps (Fig. 11b) specific shell elements with 4 nodes have been used for combining the membrane behavior with that of a flexible plate.

Table 3: Materials’ properties by Eurocode-6

Material	f_{wd} [MPa]	E [Mpa]	γ_s [kN/m^3]	m_s [kg/m^3]
1	0.87	1740	22	2243
2	0.66	1318	18	1835
3	n.a.	2600	18	1835

An adequate mesh was created in such a way to model the real behavior of the structural elements. The preliminary mesh was composed by 362 shell elements and 10 frames for a total of 404 nodes. In order to increase the reliability of the numerical model, the mesh was refined (Fig.11c) dividing opportunely the starting elements (the thickened model has 10541 elements and 10668 nodes).

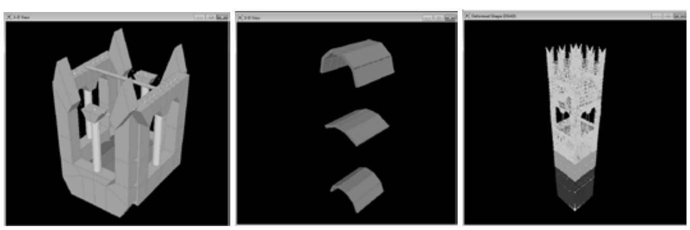

Fig.11 a) detail of the columns b) details of the vaults c) FE model.

The vaults filling material, that does not have a structural function, has been considered in the model as a load acting on the caps, while the bells have been modeled as added masses.

The weight P of the three bells, having different diameter D, has been estimated by Eq.(2) [37].

$$P = 580.8 \cdot D^{2.7839} \qquad (2)$$

Considering no inclination signs on the tower, the tower has been modelled as fixed at the base. The adjacent buildings effects have also been considered in the model, introducing some joint constraints on the lateral walls connected to the adjacent units.

The starting dynamical analysis of the FE model gives the results shown in Table 4, referred to the first 7 frequencies of the model.

For each identified mode, Table 4 reported also the excited percentage mass in the two principal direction *x* (U_x) and *y* (U_y) and the rotations around *z* axis (R_z) in such a way as to identify the typology of each mode.

Table 4: Modal frequencies and participating mass ratios of the FE model

Mode number	Frequency [Hz]	U_x	U_y	R_z
1	2.215	0.0003	0.37	0.13
2	2.393	0.33	0.0003	0.09
3	5.367	6·10(-8)	0.0012	0.15
4	8.901	10(-6)	0.24	0.1
5	11.07	0.13	1.1·10(-9)	0.04
6	12.46	4.8·10(-7)	6.5·10(-5)	1.7·10(-5)
7	14.48	1.9·10(-6)	0.011	0.011

The participating mass ratios in Table 4 and the animation of the mode shapes clearly indicate that the first and the fourth modes of the model are flexional on *y* direction, the second and the fifth are flexional on *x* direction, the third and the seventh are mainly torsional, while the sixth is a mode referred to a local movement on the *z* axis.

The model frequencies calculated in this work are closer to the identified ones with respect to the ones reported in [1], even if the same material properties have been used. This is due to the major complexity and accuracy of the FE model (Fig. 11c) and the careful observation of the architectural details. Anyway an updating procedure will also be considered for improving the quality of the model and its adherence to the identified frequencies.

In the forthcoming part of the research the model will be updated in order to match the identification results [38], the main objective of this study will be the characterization of the behavior of the cracked masonry and the interaction between the tower and the surrounding buildings.

V. Conclusions

The dynamic identification of Annunziata bell tower presented in this study is characterized by many difficulties especially for the high damaged state of the tower, the impossibility to achieve the higher floors of the structure and the presence of heavy external disturbances in environmental conditions. The tower is a historical heritage of Corfu; so it was very important and interesting to apply a non-destructive strategy to identify its dynamic behavior.

For the modal identification two different statistical approaches in different domains were used in order to evaluate the frequencies of the structure. The statistical analysis performed on the identified frequencies shows the extreme repeatability and consistency of the first six estimated frequencies with respect to all the tests and the two adopted methods. This demonstrates that, nevertheless the different casual events, the identified frequencies were persistent in all the cases.

The analysis carried out in this paper shows also that the influence of material data assumed for the calculation is relevant especially in presence of an important damage state of the building. Therefore, a considerable effort is required in defining a reliable model that could efficiently describe the identified experimental dynamical properties.

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