Enhancing Images using the Forward-Backward UFIR Algorithm


Abstract— In this paper, we present a novel implementation of the Unbiased FIR (UFIR) filter to enhance binary images for computer vision applications. The Forward-Backward (FB) algorithm is implemented to enhance piecewise-smooth images. Furthermore, we present several computer simulation examples where geometrical binary images are used to illustrate the outstanding usefulness of this method. Finally, the results are evaluated in terms of the root mean square errors (RMSEs) and the signal-to-noise ratio (SNR).

Keywords— Binary Images, Computer Vision, Forward-Backward Filtering, Unbiased FIR Filter.

I. INTRODUCTION

The problem of saving a sharp edges with a simultaneously enhancing the image, is typical in image processing. An overall panorama of nonlinear filtering that follows the median strategy has been given by Pitas and Venetsanopoulos in [1], including the important modifications for a large class of nonlinear filters employing order statistics. This type of nonlinear filters is applied in a variety of areas such as biomedical images processing [2], and remote sensing [3]. Currently, computer vision is a powerful tool for feature extraction and image processing [4], [5] and is used in many systems for robots development. Human vision is a sophisticated system that senses and reacts on visual stimuli; computer vision is intended to mimic the same function. The purpose of both systems is to interpret spatial data, data that have more than one dimension. Even though computer and human vision are functionally similar, you cannot expect that a computer vision system would be able to replicate exactly the function of the human sight. It is well known that a computer image is a matrix of pixels; the value of each pixel is a sample of the brightness of the corresponding points in the scene. The matrix of pixels, that is to say the image, is usually square and an image may be describe as $N \times N$ m-bit pixels, where $N$ is the number of points and $m$ is related to the number of brightness values. Using $m$ bits values, ranging from 0 to $2^m - 1$ are obtained. A binary image is a digital image that has only two possible values for each pixel ($m = 1$). Typically the two colors used for a binary image are black and white. Binary images often arise in digital image processing as masks or as the result of certain operations such as segmentation, threshold, and dithering [5], [6]. Typical applications of the binary images are for example in the analysis of documents, industrial applications for the recognition of objects, as a priori information in complex algorithms, among others.

Besides, in off-line processing, forward-backward (FB) filters and smoothers are often used. There are two typical rules of how apply and combined forward and backward estimates and thus implement and connect both the causal and non-causal filter structures. The first way implies conducting forward and backward processing simultaneously and the combine the results at the output [7], [9]. Another way is to process a signal forward and then repeat it backward. Denoising is commonly more efficient here, although the procedure requires extra time [8]. The approach can be implemented using the time-variant FIR smoothers, which are able to unite advantages of linear filtering with nonlinear filtering robustness. Moreover, the UFIR filter is a new family of orthogonal polynomials [10] and was originally derived by Shmaliy [11] to have unique coefficients of the polynomial FIR function for the same degree. The filter was then developed to be $p$-shift with filtering ($p=0$), $|p|\text{-lag}$ smoothing ($p<0$) [11], [12], and $p$-step prediction ($p>0$) [11] properties. Thus, in this paper we use the FB algorithm with UFIR filters to enhance binary images.

II. MAIN IDEA OF FB-ALGORITHM AND UFIR SMOOTHER

The main idea behind the solution proposed is illustrated in Fig 1. Suppose that a discrete-time piecewise smooth signal $x_n$ (dashed) is measured in the presence of noise as $y_n$ and the breakpoints $k_q, q=0, 1, \ldots$ are known. To filter noise out, the following algorithm can be used [13]:

...
1. Smooth measurement Forward by the UFIR smoother [9] from \( k_0 + 1 = 0 \) to \( n = k_1 \). If noise is White Gaussian with variance \( \sigma^2_{\text{Forward}} = \sigma^2 / (k_1 + 1) \). Next, smooth the result Backward, from \( k_1 \) to \( k_0 + 1 \). The Backward estimate variance \( \sigma^2_{\text{Backward}} \) at \( k_0 + 1 \) can be expected to be a bit larger than \( \sigma^2_{\text{Forward}} \) but much smaller than \( \sigma^2 \).

2. Next, smooth the result in the same manner Backward. The Backward estimate variance can be expected to be nearly the same, \( \sigma^2_{\text{Backward}} = \sigma^2 / (k_1 + 1) \).

3. Pass across the edge, from \( k_1 \) to \( k_1 + 1 \), with the ramp UFIR filter on the window of \( N = 2 \) points.

4. Repeat this procedure for all segments of the piecewise signal from \( k_q + 1 \) to \( k_{q+1} \).

**A. Polynomial Smoother Impulse Response**

Linear FB denoising algorithm can be implemented out based on the time-variant UFIR structures if we first modify the smoothing UFIR filter proposed in [12]. The relevant modification is discussed below. Suppose that a piecewise-smooth signal (or image) \( x_n \) is represented from \( k_0 + 1 \) to \( k_1 \) (Fig. 1) with the finite Taylor series of order \( K - 1 \) as follows [15], [14]:

\[
x_n = b_0 + b_1 \tau n + b_2 (\tau n)^2 / 2 + \cdots + b_{K-1} (\tau n)^{K-1} / (K-1)!,
\]

where, \( \tau = t_n - t_{n-1} \) is sampling time and \( b_r, r \in [0, K - 1] \), is some constant coefficient. By a time-shift (1) can be applied to any smooth part of \( x_n \) from \( k_q + 1 \) to \( k_{q+1} \). Assume that \( x_n \) is measured in the presence of zero mean AWGN \( v_n \) as

\[
y_n = x_n + v_n.
\]

If we introduce \( h_0(N, p) \) as the gain of \( l \)-degree polynomial \( p \)-step dependent filter, then the estimate of the electronic image \( \hat{x}_n \) can be obtained based on averaging concept by the convolution on horizon of \( N \) points

\[
x_n = \sum_{m=-p}^{N-1-p} h_i(N, p) y_{n-m}
\]

where the positive step, \( p > 0 \), is supposed for predictive FIR filtering, \( p = 0 \) for FIR filtering, and \( p < 0 \) for smoothing FIR filtering. Measurement \( y_n \) can be smoothed on an interval of \( N \geq k_{q+1} - k_q - 1 \) points, with the polynomial impulse response \( [p]-\text{lag} \), represented in the forms of

\[
\tilde{h}_0(N, p) = \sum_{j=0}^{l} a_{j}(N,p)(i+p)^{j}
\]

and

\[
\tilde{h}_l(N, p) = \sum_{j=0}^{l} a_{j,l}(N,p)i^{j},
\]

where \( l = K - 1 \) and the coefficients \( a_{j}(N,p) \) is specified by

\[
a_{j}(n,p) = (-1) \frac{M(j+1)(Np)}{|D(N,p)|},
\]

via a short \( (l + 1) \times (l + 1) \) Hankel matrix

\[
D(N,p) = \begin{bmatrix} d_0(N,p) & d_1(N,p) & \cdots & d_n(N,p) \\ d_1(N,p) & d_2(N,p) & \cdots & d_{n+1}(N,p) \\ \vdots & \vdots & \ddots & \vdots \\ d_n(N,p) & d_{n+1}(N,p) & \cdots & d_{2n}(N,p) \end{bmatrix},
\]

each component in (6) can be found using the recursive relation of the Bernoulli polynomials \( B_n(x) \) established in [11], [12]. Note that, so far, the following low-degree polynomials, \( h_0(N,p) \), were found and investigated:

A model that is uniform over an averaging horizon of \( N \) points is the simplest one. The relevant signal is characterized with one state and the filter gain is represented by (3), with the 0-degree polynomial as

\[
h_0(p) = \begin{cases} \frac{1}{N} & p \leq N - 1 + p \\ 0 & \text{otherwise} \end{cases}
\]

For linear models, the \( p \)-lag gain, existing from \( p \) to \( N - 1 + p \), becomes ramp

\[
h_l(p) = a_{0,l}(p) + a_{1,l}(p)
\]

having the coefficients

\[
a_{0,l}(p) = \frac{2(2N-1)(N-1) + 2p(N-1+p)}{N(N^2-1)}
\]

and

\[
a_{1,l}(p) = \frac{6(N-1+2p)}{N(N^2-1)}
\]

For the quadratic and cubic models, the gain of the unbiased smoothing FIR filters are shown in [11] and [12].

**B. FB-UFIR denoising Algorithms**

Follow the [14]; the operation principle of the FB UFIR smoothing algorithm is development using the Forward (FW) and Backward (BW) structures. First, consider if a binary
image is considered as a signal in 2D, and this is composed of bar pieces, the FB UFIR smoothing algorithm can be developed as follow:

The FW smoother output can thus be computed starting with \( k_0 + 1 \) and finishing at \( k_1 \) using the follow algorithm

\[
\mathcal{X}_{k_0 + p | k_1}^{FW} = \sum_{m=0}^{k_1-k_0-1} \mathcal{H}_m (k_1 - k_0, p) y_{k_1-m} \quad \text{(11)}
\]

if \( k_0 + 1 \leq n \leq k_0 + 1 + l \), set \( \mathcal{X}_{n|k_1} = y_n \).

The output provided in such a way that is further inverted in time as the procedure (11) applied once again in order to smooth Backward (BW) as follow

\[
\hat{\mathcal{X}}_{n|k_1}^{BW} = \sum_{m=0}^{k_1-k_0-1} \mathcal{H}_m (k_1 - k_0, p) \hat{\mathcal{X}}_{k_1-m|k_1}^{FW}
\]

(12)

If \( k_0 + 1 \leq n \leq k_0 + 1 + l \), set \( \hat{\mathcal{X}}_{n|k_1}^{BW} = \mathcal{X}_{n|k_1}^{FW} \). The FB estimate of the first smooth part is now formed the time-reversal of

\[
\mathcal{X}_{n|k_1}^{FW} = \hat{\mathcal{X}}_{n|k_1}^{FW} = \mathcal{X}_{n|k_1}^{BW} = \mathcal{X}_{k_0+k_1+1-n|k_1}^{BW}
\]

(13)

The image edge following after \( k_1 \) is viewed as a linear function, from \( k_1 \) to \( k_1 + 1 \). Thus, the UFIR filter passes across the edge with the ramp impulse having \( N = 2 \) and \( p = 0 \). Denoising the subsequent smooth parts can be accomplished in the same manner. Just substitute in (12)-(13) \( k_0 \) with \( k_q \) and \( k_1 \) by \( k_{q+1} \). The FB estimate of the piecewise image will thus be

\[
\mathcal{X}_{n|q+1}^{FW} = \hat{\mathcal{X}}_{n|q+1}^{FW} = \mathcal{X}_{n|q+1}^{BW} = \mathcal{X}_{k_0+k_{q+1}+1-n|k_{q+1}}^{BW}
\]

(14)

and the output taken from \( k_q + 1 \) to \( k_{q+1} \).

### III. SIMULATIONS

In this section, we present some computational evaluations to verify the improvement of FB UFIR algorithm. We employed two special cases, in the first case; we use a square pulse signal of 512 samples and for the second case, an artificial image of 512 by 512 pixels. All evaluation is carried out using a personal computer that used an Intel CORE 2 DUO processor with 2.4GHz of speed and 2GB of RAM.

In Fig. 2, we showed the original and noisy signals, respectively. The original signal is a square pulse. This signal is contaminated with a variance of 0.1 of white noise. The approximation order of \( p \)-UFIR filters is lineal and quadratic with \( N = 11 \). The horizon is calculated as shown in [9] and [11]. The values of horizon in each case are, \( p = -5 \) and \( -8 \) to the linear and quadratic cases, respectively. In Fig. 3 and 4 we can see the enhanced square pulse using the FB \( p \)-UFIR algorithm. The quantitative evaluations are reported in the Table I. We can see that the linear impulse response performs best in the sense of Signal-to-Noise Ratio (SNR) and Root-Mean-Square-Error (RMSE), respectively.

<table>
<thead>
<tr>
<th>( h(x,N,p) )</th>
<th>RMSE</th>
<th>SNR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_1 )</td>
<td>0.0415</td>
<td>22.9017</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>0.0442</td>
<td>22.3561</td>
</tr>
</tbody>
</table>

Fig. 2 shows the original test image and the blurred image or measurement image is contaminated with Gaussian noise (AWGN) based on (2). The initial conditions for filtering the image are the same as used for the signals. Thus, consider two case studies; with variance of 0.1 and 0.2. Both images are shown in Fig. 6 and 7, respectively.
For the proper operation of the FB algorithm, it is important to establish spatially, the location of the edges throughout the image. To perform this process, we have used the Prewitt edge detector [6]. The use of this edge detector is based on that detection can be performed in vertical and horizontal edges. The edge detection is shown in Fig. 8 and 9, respectively. The threshold of edge detector was set at $th = 0.2$.

In Fig. 10 and Fig. 11 we had shown the enhanced binary images using the FB $p$-UFIR algorithm. The quantitative evaluation of image processing is present in the Table II and III, respectively. Finally, the quantitative evaluations are carrying out using the Signal-to-Noise Ratio (SNR) and the Root Mean Square Error (RMSE). Both metrics are employed in determining the performance of the FB algorithm.
with the impulse response of the linear I, II and III show the performance obtained by the algorithm the detection of the edges in the image. Finally, in the Tables some lines in both enhanced images are due to errors during Fig. 10 and 11 to binary images, respectively. We can see that breakpoints detected using the Prewitt edge detector.

This article presented a suitable way to derive p-UFIR filters focused to the enhancing images. The FB algorithm have been employed for piecewise-smooth images with breakpoints detected using the Prewitt edge detector. Qualitative results are shown in Fig. 4 and 5 to signals and Fig. 10 and 11 to binary images, respectively. We can see that some lines in both enhanced images are due to errors during the detection of the edges in the image. Finally, in the Tables I, II and III show the performance obtained by the algorithm with the impulse response of the linear p-UFIR filter, $h_p(N, p)$.

**IV. CONCLUSION**

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**REFERENCES**


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