Modelling and Cost Estimation of Stirling Engine for CHP Applications

Ana C. Ferreira, Ricardo F. Oliveira, Manuel L. Nunes, Luís B. Martins, and Senhorinha F. Teixeira

***Abstract*—**Thermodynamic analysis of the Stirling engine's performance has been conducted, using specially designed computing codes along with the thermal balance study of the technology. The performance of the unit has been evaluated considering different operational conditions, which include the electrical and thermal production, working fluid mean pressure or mass, components geometrical sizing. The thermal-economic evaluation represents an effective tool to optimize a power plant with this type of technology. This study presents a mathematical model that includes a set of equations able to describe and simulate the physical system, as well as a set of equations that define the cost of each plant component. This paper presents a numerical study faithfully simulating the real conditions of a micro-CHP unit based on an alpha type Stirling Engine. The simulations were performed through a MatLab® code that assesses the thermodynamic efficiency, including heat transfer limitations and pumping losses throughout the system. Results showed the Stirling engine performance depends on geometrical and physical parameters which optimization is required in order to obtain the best performance. It is verified that cost estimation based on sizing and quality parameters has a good correlation with the capital investment costs of commercial models.

***Keywords*—**CHP Applications, High-efficient energy conversion, Stirling engines, Cost estimation.

I. Introduction

CURRENTLY, there is a strong pressure for the development of energy systems able to deliver a less pollutant and more efficient energy conversion process. The growing worldwide demand for those energy conversion systems has led to a renewed attention in the use of cogeneration technologies. The concept of micro-Combined Heat and Power (µ-CHP) or micro cogeneration has been a known for long time. Cogeneration systems have the ability to produce both useful thermal energy and electricity from a single source of fuel. Concerning to this framework, great interest has been shown in low power systems able to deliver an energy output of 1-10 kW_e [1], [2]. These power plants are specially designed to cover fairly higher heat loads, which appears to be a good opportunity to meet the global energy needs of households [3].

Stirling Engines (SE) technology is based on an external combustion or other external heat-source, thus allowing the use of different primary energy sources including fossil fuels (oil derived or natural gas) and renewable energies (solar or biomass). In these engines, the working gas (e.g. helium, hydrogen, nitrogen or air) operates on a closed regenerative thermodynamic cycle, with cyclic compression and expansion of the working gas at different temperature levels [4]. Despite the fact that this technology is not widely used, there is a proved interest on Stirling engines because of their high-global efficiency, good performance at partial load, fuel flexibility and low gas and noise emission levels [5]. Stirling engines have the potential of achieving higher efficiencies because they closely approach the Carnot cycle. Presently, these engines are able to get an electrical efficiency of about 30% and a total efficiency of 85-98% (based on Low Heating Value, LHV) operating in cogeneration mode [6]. Commonly, the potential heat sources for SE operation are fossil fuels and solar energy. Nevertheless, more recently, several recent practical applications uses biomass or the waste heat as fuel [7]. Also, the nature of external combustion means that there is no transient combustion or mechanical valves; therefore with careful technical design, a low-noise and low-vibration system can be achieved. The SE is also characterized by longer operational lifetimes when compared with internal combustion engines.

Nevertheless, there are some limitations for SE technologies. Some components of the engine should be manufactured with special alloys because of the high temperature and pressure operational conditions endured by the system. This increases the production costs requiring, therefore, high investment costs. Plus, the choice of the "ideal" gas can bring some difficulties associated with its

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ability to diffuse through materials, which works at high operation pressures.

Despite these limitative aspects of SE, this technology fulfils a number of requirements for thermal applications. Table I presents the actual energy requirements and the attractive features of the SE technology.

TABLE I. POWER PLANT NEEDS AND ATTRACTIVE FEATURES OF SE

Technological Needs	SE Characteristics
Reducing conventional fuels use	Flexible fuel sources
Increasing fossil fuel costs	Low fuel consumption
Use of alternative fuels	Low noise and vibrational levels
Reduction of gas emissions	Clean combustion
Waste heat recovery	High thermal efficiency

The Stirling engine performance depends on geometrical and physical features of the engine and on the working fluid properties, such as regenerator effectiveness, engine swept and dead volumes or the temperature of heat sources. Several studies have been reported in the literature concerning to the study of SE optimization for small and micro-scale applications. Puech and Tishkova [8] performed a thermodynamic analysis of a Stirling engine conducting an investigation about the influence of regenerator dead volume. The results showed that the dead volume amplifies the imperfect regeneration effect. Boucher et al. [9] related a theoretical study of the dynamic behaviour of a dual free-piston Stirling engine coupled with an asynchronous linear alternator. The objective was the evaluation of the thermo-mechanical conditions for a stable operation of the engine. Formosa and Despesse [10] developed an analytical thermodynamic model to study a free-piston Stirling engine architecture. The model integrated the analysis of the regenerator efficiency and conduction losses, the pressure drops and the heat exchangers effectiveness. The model was validated using the whole range of the experimental data available from the General Motor GPU-3 Stirling engine prototype. The influence of the technological and operating parameters on Stirling engine performance was investigated. The results from the simplified model and the data from the experiment showed a reasonable correlation. Rogdakis et al. [1] studied a Solo Stirling Engine V161 cogeneration module via a thermodynamic analysis. Calculations were conducted using different operational conditions concerning the heat load of the engine and the produced electrical power. The authors achieved good results in terms of electrical and thermal efficiencies as well as a positive primary energy saving.

Asnaghi et al. [11] also performed a numerical simulation and thermodynamic analysis of SOLO 161 Solar Stirling engine. He and his co-authors considered several imperfect working conditions, pistons' dead volumes, and work losses in the simulation process. According to their studies, regenerator effectiveness, heater and cooler temperatures, working gas, phase difference, average engine pressure, and dead volumes are parameters that affect Stirling engine performance, which was estimated for different input considerations. Also, the results indicated that the increase in the heater and cooler temperature difference and the decrease in the dead volumes will lead to an increase in thermal efficiency. Kongtragool and Wongwises [12] investigated the effect of regenerator effectiveness and dead volume on the engine network; heat input and efficiency by using a theoretical investigation on the thermodynamic analysis of a Stirling engine.

Besides the thermodynamic efficiency evaluation, it is of utmost importance to integrate technical variables and the costs in order to simultaneously optimize the physical and the economic output.

Thermal-economics combines thermodynamic analysis and economic principles, in each of power plant components, in order to evaluate the costs of energy production. These analysis can be useful to disclose the most cost-effective components and thus in improvement of the overall system design [13]. This type of evaluation requires the definition of a thermodynamic and economic model of the system [14]. In literature, different studies can be found concerning the thermal-economic optimization of power plants for CHP applications, that apply different methodologies and approaches [13], [15]–[19].

The main objective of this study is to analyse an alpha Stirling engine (non-ideal analysis), by trying to disclose the best operational parameters for a SE engine for cogeneration applications. The paper presents a mathematical model able to describe the physical system, using a software-code developed in the MatLab® environment. The physical model was based on the work of Urieli and Berchowitz [20] while the cost equations were developed based on a methodology also applied in gas turbines optimization [14]. Cost equations depend on physical parameters that also affect the thermodynamic performance of the engine and the solution will be the combination of parameter values that will lead to the best economic output. The system costs are estimated for the best physical model result.

II. SYSTEM DESCRIPTION

This study is focused on the optimization of a CHP system based on a Stirling engine as prime mover, able to produce 1-5 kW of electricity and a larger heat load considering a heat-to-power ratio of about 3.5 to 5.0. This case study pretends to analyse a system able to fulfil the energy requirements of a residential dwelling. Accordingly to the literature [3], for individual dwellings, most decentralized energy systems are characterized by a thermal power output in the range of 2-35 $kW_{th.}$

The chosen alpha-Stirling configuration consists of two mechanically linked pistons located in separate cylinders that define the compression and expansion spaces. The working gas flows between these two spaces by alternate crossing of, a low temperature heat exchanger (cooler), a regenerator and a high temperature heat exchanger (heater), connected in series.

Thus, the engine is considered as a set of five connected components, consisting of the compression space (*c*), cooler (*k*), regenerator (*r*), heater (*h*) and the expansion space (*e*). Each engine component represents an entity endowed with its respective volume (*V*), temperature (*T*), absolute pressure (*p*) and mass (*m*). Fig. 1 shows a general representation of a SE with thermal interfaces.

Heat is transferred from the external heat source to the working gas in the heater, cyclically stored and recovered in the regenerator, and rejected by the working gas in the cooler. Thus, Stirling engine works between two temperatures T_h and T_c (hot and cold sink, respectively). These temperatures correspond to the values of 725 K and 353 K, respectively. The value for the hot temperature was assumed considering that the energy source is concentred solar radiation. The cold sink of the engine is refrigerated by a mass flow of water which removes heat from the cooler to produce hot water. It was assumed that the mass flow of water is heated from 288 K to 343 K.

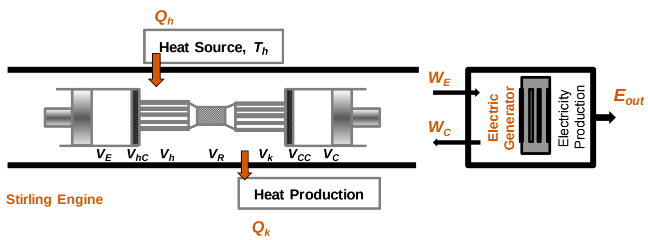

Fig. 1 Representation of a general Stirling Engine.

Several input parameters are required for the analysis. The mass of the operating gas is determined by the Schmidt analysis [21], which requires some design parameters: mean operating pressure (p_{mean}), cylinder swept volumes, clearance volumes, hot and cold temperatures.

Three working fluids have been previously studied [22]: air, helium and hydrogen. Hydrogen and helium were selected here to perform the numerical simulations due to their best results in terms of engine efficiency [23]. On one hand, the working gas should have a high thermal conductivity, for improved efficacy of the heat exchangers, and a low density and specific heat capacity so that a given amount of heat leads to a larger increase in pressure or volume. The combined property is the thermal diffusivity (thermal conductivity divided by density and heat capacity) and helium is the best option. On the other hand, a lower dynamic viscosity and density reduces the pumping losses, improving engine specific power and efficiency. Hydrogen scores better here, primarily because the engine can run at higher speeds [22].

III. Mathematical Model

A. Physical Model

The mathematical model in the present works integrates two analyses to reach the numerical solution of the thermodynamic cycle. Firstly, it invokes an Ideal Adiabatic simulation and, sequentially, a Non-ideal simulation that evaluate the heat transfer and pressure loss effects in the Stirling engine [22].

In the adiabatic analysis, the model is treated as a "quasi steady-flow" system. A set of ordinary differential equations is iteratively solved, considering an initial-value problem in which the initial values of all the variables are known and the equations are integrated from that initial state over a complete engine cycle. The final state of the cycle is then used as a new initial-value for a new cycle and several iterations are made until cycle convergence is obtained. The resulting equations are linked by applying the mass and energy equations across the entire system. Enthalpy is transported by means of mass flow and temperature at each component. In the Ideal Adiabatic analysis, the energy equation can be written as in (1)

$$dQ + C_p T_{in} dm_{in} - C_p T_{out} dm_{out} = dW + C_v d\left(mT\right) \quad (1)$$

where the derivative operator is denoted by *d*, thus for example *dm* refers to the mass derivative $dm/d\theta$, where θ is crank the cycle angle.

After running the Ideal adiabatic analysis, the numerical process follows with a Non-Ideal simulation which includes the effects of non-perfect regeneration and the pumping losses. The term “pumping loss” refers to the work required to press on the working gas through the heat exchangers and regenerator, thus reducing the net power output of the engine. The non-ideal effects of the regeneration are mainly due to the convective thermal resistance between the gas and the regenerator surface, and can be modelled by using the Number of Transfer Units (NTU) method, with NTU defined in terms of a Stanton number (St), as in (2):

$$NTU = \mathrm{St} \cdot \left(A_w / A \right) \cdot \frac{1}{2} \quad (2)$$

where A_w refers to the wall/gas, or "wetted" area of the heat exchanger surface and A is the free flow area through the matrix. The St can be defined as in (3):

$$\mathrm{St} = \frac{h}{\rho u c_p} \quad (3)$$

where *h* is the convective heat transfer coefficient, ρ is the gas density, *u* is the velocity and c_p is the specific heat capacity of the gas. The factor 2 in Eq. (2) is due to the fact that St is usually defined for heat transfer from a gas stream to a wall, whereas in the cyclic process of the regenerator, heat is also transferred from the matrix to the gas flow. In the “loading” process, the hot working gas is pre-cooled, while flowing through the regenerator from the heater to the cooler, transferring heat to the regenerator matrix. Then, in the reverse process, the heat that was previously stored in the matrix is “discharged” and pre-heats the cold gas that flows into the heater and expansion space. The regenerator effectiveness for Stirling engines can be defined as the ratio between the real amount of heat transferred from the matrix to the working fluid and the maximum possible amount of pre-heating used in the ideal adiabatic model. The regenerator effectiveness can be obtained by (4).

$$\varepsilon_r = \frac{NTU}{1+NTU} \tag{4}$$

The effectiveness of the heater and cooler can also be evaluated by NTU method, considering constraint wall temperatures. The heat exchanger effectiveness for both exchangers are defined as in (5).

$$\varepsilon = 1 - e^{-NTU} \tag{5}$$

The mean effective temperatures in the heater (T_h) and the cooler (T_k) are, respectively, lower and higher than the corresponding heat exchanger wall temperatures, heater ($T_{h,w}$) and cooler ($T_{k,w}$). This implies that the engine is operating between lower temperature limits than originally specified which effectively reduces the thermodynamic engine efficiency (see Fig. 2).

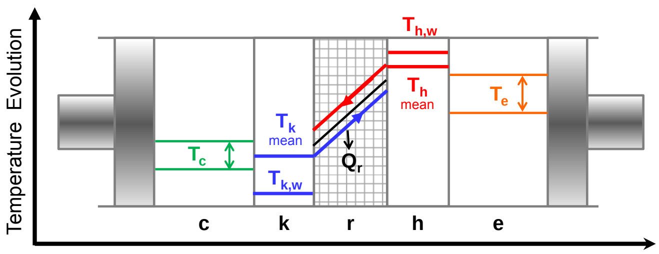

Fig. 2 Evolution of heater, cooler and regenerator effective temperatures for non-ideal analysis.

So, the total heat transfer can be calculated as in (6):

$$Q = h \cdot A_w \left(T_w - T\right) \tag{6}$$

where T_w is the wall temperature, and T the mean effective gas temperature (heater or cooler). As the temperatures are determined iteratively, Q_h and Q_k can be evaluated and the regenerator heat transfer reduction, Q_{rloss}, is quantified in terms of the regenerator effectiveness. Thus, the reduction of heat transfer in the regenerator can be quantified as in (7).

$$Q_{rloss} = \left(1 - \varepsilon_r\right) \cdot Q_{r,ideal} \tag{7}$$

The less heat transfer in the regenerator leads to increases in the heats of the hot and cold sources so that, the heat for both heat exchangers is determined in (8) and (9), respectively.

$$Q_k = Q_{k,ideal} + Q_{rloss} \tag{8}$$

$$Q_h = Q_{h,ideal} + Q_{rloss} \tag{9}$$

Fluid friction associated with the flow through the heat exchangers, results in a pressure drop, reducing the output power of the engine. The pressure drop (ΔP) is taken over the three heat exchangers and then, the value of the corresponding work can be achieved by integration over the complete cycle. The total engine work per cycle, W is given by (10):

$$W = \oint P\left(dV_e + dV_c\right) - \oint \sum \Delta P dV_e \tag{10}$$

where V_e and V_c are, respectively, the expansion and compression volumes. The first term in the equation represents the ideal adiabatic work done per cycle and the second one represents the pressure drop per cycle, ΔW, converted to work loss, as in (11):

$$\Delta W = \int_0^{2\pi} \left(\sum_{i=1}^{3} \Delta P_i \frac{dV_e}{d\theta} \right) \cdot d\theta \tag{11}$$

where θ is the crank angle. The pressure drop is evaluated by (12).

$$\Delta P = \frac{-2C_f \operatorname{Re} \mu u V}{d^2 A} \tag{12}$$

where C_f is the friction coefficient, Re is the Reynolds number, μ and u are the gas viscosity and velocity, d is the hydraulic diameter of the small parallel passages and V the void volume. The friction coefficient is the non-dimensional wall shear-stress, calculated according to (13):

$$C_f = \frac{\tau}{0.5 \rho u^2} \tag{13}$$

where τ, is the wall shear stress, ρ *is* the working fluid density and u is velocity. The choice of working gas is typically made, considering the gas that allows the best efficiencies.

B. *Cost Estimation of the Thermal Components*

The mathematical expressions that define the cost of each component were based on the methodology developed by Marechal et al. [24]. The costing methodology consists on a derivation of an expression for each component by integrating thermodynamic and cost coefficients adjusted for this kind of technology and also taking into account real market data. Each cost equation was defined considering some of the physical variables that integrate the thermodynamic model. These variables can be divided in size and quality variables.

In terms of methodology, the equations were defined considering that the cost of each component of the system, being defined a purchase cost equation representative of each component. Thus, the cost equation was defined according to: $C = C_{ref} \cdot F_m \cdot F_c$.

The term C_{ref} is the reference cost coefficient which corresponds to a cost per unit of (one or more than one) physical parameter. The term F_m is the sizing factor which scales the system component from a reference case, as presented in (14):

$$F_m = F_{ref} \cdot \left(\frac{F_i}{F_{ref}} \right)^b \tag{14}$$

where F_{ref} and F_i are the reference and the physical variable value and b the sizing exponent. Due to the high temperatures at which certain system components operates, an additional term can be included into the cost component equation. The temperature factor, F_T, can be defined as in (15).

$$F_T = \frac{1 + e^{C_i \cdot (T_i - T_{ref})}}{2} \tag{15}$$

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This temperature factor was included into the purchase cost equation of the heater and the regenerator. The temperature factor is defined considering a constant, C_i, the reference and the effective temperature, T_i and T_{ref}, respectively.

As a result, the cost estimation can be performed in order to evaluate the overall cost of the system considering the cost share of each component. The purchase cost equations for heater, regenerator and cooler are presented by: (16), (17) and (18), respectively.

$$C_h = C_{11,h} \cdot A_{ref,wh} \cdot \left(\frac{A_{wh}}{A_{ref,wh}}\right)^{0.5} \cdot \left[\frac{1+e^{C_{12,h}\cdot(T_h-725)}}{2}\right] \quad (16)$$

$$C_r = C_{21,r} \cdot A_{ref,wr} \cdot \left(\frac{A_{wr}}{A_{ref,wr}}\right)^{0.6} \cdot \left[\frac{1+e^{C_{22,r}\cdot(T_{reg}-600)}}{2}\right] \quad (17)$$

$$C_k = C_{31,k} \cdot A_{ref,wk} \cdot \left(\frac{A_{wk}}{A_{ref,wk}}\right)^{0.4} \quad (18)$$

For the heater, regenerator and cooler, the equations relate the cost of the exchanger with its effective heat transfer area. In the specific case of the heater and regenerator, an additional correction term must be added in order to include the temperature effect in their cost. The design of these two special heat exchangers is affected by working fluid temperatures, pressures and the type of heat source. For instance, the flow at heater's outer surface is characterized by a high temperature, low-pressure steady conditions, while, in the internal surface, the fluid flows at high temperature and high pressure, subjected to turbulence. These constraints make these thermal components more expensive because of the materials used in their manufacture. Several correlations must be done before assuming the cost coefficients or the sizing exponents. The cost of each component is estimated considering a reference case from the available market. Reference values for the heat transfer area and cost coefficients are assumed by estimating their share in the total capital cost. Fig. 4 presents the regenerator cost estimation considering three different sizing exponents. The regenerator can be considered the heart of the Stirling engine; thus, adequate materials have to be used in its manufacturing because this special heat exchanger is responsible for the critical temperature changes in the working fluid. The heat transfer was calculated assuming that the regenerator has a fine wired matrix in order to improve the heat transfer process by exposing the maximum surface area of the matrix. Therefore, parameters such as the matrix porosity and the wire diameter are important to optimize in the design of the regenerator.

A cost equation representative of the engine body must be also included. Thus, the engine bulk cost equation can be defined as in (19).

$$C_{eng} = C_{41,eng}\left[V_{ref,eng}\left(\frac{V_{eng}}{V_{ref,eng}}\right)^{0.2} \cdot P_{ref,eng}\left(\frac{P_{eng}}{P_{ref,eng}}\right)^{0.2}\right] \quad (19)$$

The power of Stirling engines is affected by changing the operational parameters such as the pressure, phase angle, volume and speed.

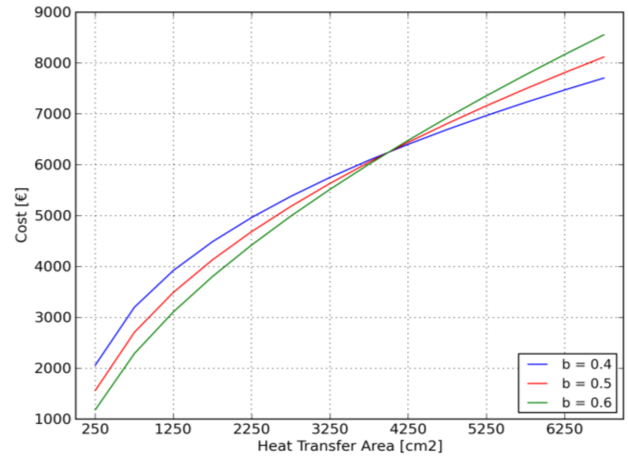

Fig. 4 Regenerator cost estimation.

Because of the complexity of system modelling, the engine bulk cost equation was estimated considering the volume and the mean pressure as the main relevant physical parameters in cost definition. Engine bulk cost includes the mean operational pressure of the system because of the proportionality between the mean pressure and the power output. Higher pressures also mean higher material costs and the need for better sealing solutions. Fig. 5 presents the engine bulk cost estimation as a function of the volume and mean operational pressure.

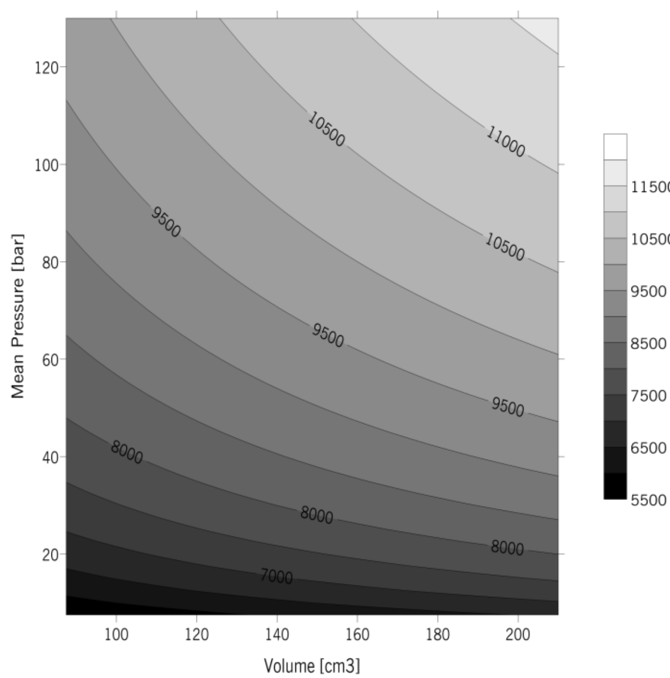

Fig. 5 Engine bulk cost estimation.

There is a significant variation on costs for different sizing parameters. Its choice must take into account the application and the power output of the thermal engine. Similar approaches were performed for the remaining components.

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IV. Results and Discussion

A. Physical Model

The simulations were carried out by running a non-ideal analysis which accounts for the pumping losses and the effects of non-perfect regeneration. Engine's geometric characteristics for the cooler, regenerator and heater, as well as the cylinders volumes and other operating input parameters were defined as in table II. Simulations were carried out considering the heater and the cooler as smooth pipes and the regenerator a wired matrix. The geometric input parameters for the heat exchangers were already presented in [22].

TABLE II. Input Parameters for Simulations

Parameter, value
Cooler volume V_k , 106 cm^3
Heater volume V_h, 84.8 cm^3
Regenerator Volume V_r, 69.8 cm^3
Cylinders Swept volumes, 130 cm^3
Cylinders Clearance volumes, 25 cm^3
Phase angle, 90º
Engine speed, 1500 rpm
Tested working gases, He, H_2
Tested mean pressure, 30 bar

Fig. 6 shows the pressure versus compression space volume diagram (P-V$_c$), the pressure versus expansion space volume diagram (P-V$_e$) and the pressure versus total space volume (P-V). This simulation was carried out considering the Helium as the working fluid.

The pressure rises during the compression phase followed by the gas pre-heating phase, where it gets to its maximum value. The minimum pressure occurs in the reverse process, when the working gas is pre-cooled and the volume is at maximum, after the gas has been expanded. The regenerator pre-heating and pre-cooling phases are not exactly isochoric due to the sinusoidal volume variations of the two pistons.

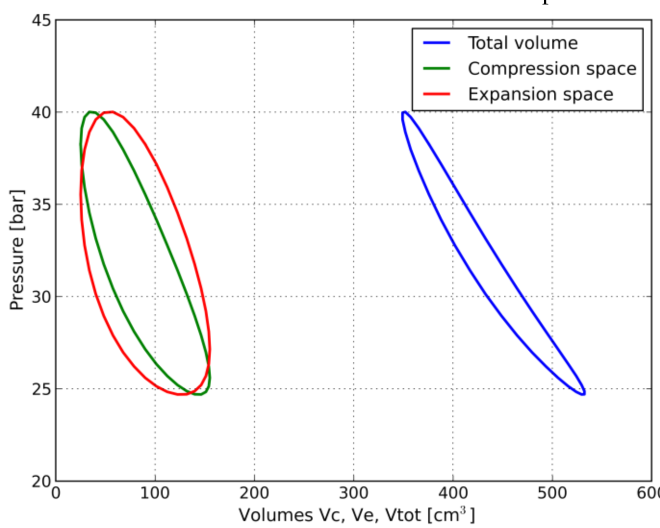

Fig. 6 Pressure versus space volume diagrams for p_{mean} = 30 bar.

Figure 7 presents the temperature variation in the heat exchangers, expansion and compression spaces. The working gas temperature in the compression and expansion spaces fluctuate along the cycle, while a mean effective temperatures for the working gas within heater and cooler is calculated.

Results show that mean effective temperatures in heater and cooler are, respectively, lower and higher than the heat exchanger wall temperatures (671.4 K and 404.2 K, correspondingly). It is also found that temperature at the expansion space could exceed the hot gas temperature and that the temperature at compression space could be less than the cold fluid mean temperature, which could be explained by compression and expansion processes in the adjacent cylinders.

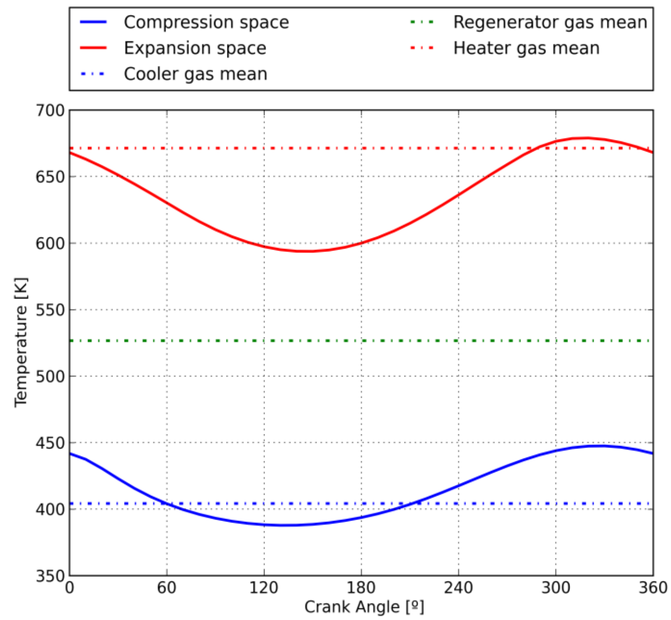

Fig. 7 Temperature variation in the heat exchangers, expansion and compression spaces, for p_{mean} = 30 bar.

The transferred thermal energy and the total work output over an ideal adiabatic cycle are shown in Fig. 8.

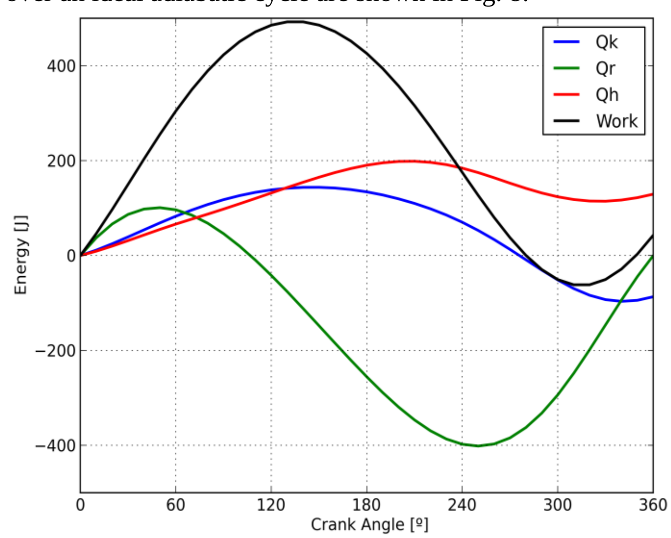

Fig. 8 Energy variation diagram for a p_{mean} = 30 bar.

The maximum heat transferred to and from the regenerator matrix is higher than the energy transferred to heater or to the cooler. This reveals the importance of this SE component, since a loss in the heats transferred by the regenerator in the pre-heating and pre-cooling processes leads to an increase in the hot and cold source and thus to a significant decrease in engine efficiency. The maximum total work reached over a complete cycle is 492.4 J for a mean pressure of 30 bar. Work is proportional to the mean pressure, which rise leads to an increase of total work.

Table III presents the results corresponding to a simulation performed at 1500 rpm and 30 bar, comparing helium and hydrogen performance.

TABLE III. NON-IDEAL SIMULATIONS FOR HYDROGEN AND HELIUM

Engine Speed (1500 rpm)	He	H_2
Hot source heat, Q_h (J)	164.8	176.3
Cold source heat, Q_k (J)	118.1	124.4
Regenerator reduction, Q_{rloss} (J)	30.26	45.36
Work (J)	46.96	52.08
Power (kW)	1.17	1.30
Efficiency (%)	28.5	29.6

Despite small, hydrogen presents a better output in terms of engine thermal efficiency, 29.6% against 28.5% for the helium. Also, power output is greater for the engine working with the hydrogen. Nevertheless, the energy reduction at the regenerator reaches higher values.

The working gas suffer friction when flowing through the heat exchangers. This effect results into pressure drop, which is higher for higher operational mean pressures and engine speed. In previous studies, it was proved that hydrogen is the working gas with lowest pressure drop. Also, the pressure drop is higher in the case of the regenerator, when compared with the heater and the cooler [22], [25].

Heat exchangers effectiveness is an important parameter for the evaluation of efficiency. Fig. 9 presents the heat exchanger effectiveness results considering helium and hydrogen at two different pressure values: 5 and 30 bar.

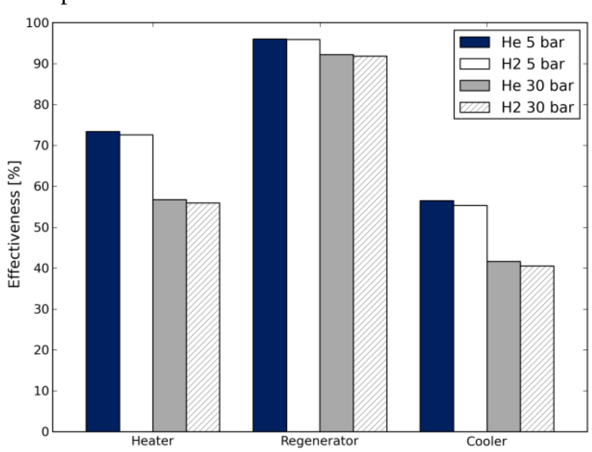

Fig. 9 Effectiveness of the heat exchangers considering helium and hydrogen as working fluids at different values of mean pressure.

The regenerator is the heat exchanger with higher effectiveness above 90% for all the tested cases, as in Fig. 9. Results also show that the heat exchangers effectiveness is slightly higher for helium when compared with hydrogen. Purely in heat transfer terms, helium is slightly better than hydrogen. Comparing the results for 5 and 30 bar, it is shown that the heat exchangers effectiveness decreases for higher values of mean pressure.

B. *Cost Estimation*

Considering the input conditions and using the calculated values from the physical model, the cost of the SE was estimated. Table IV presents the cost estimation for each system component as well as the total capital cost.

TABLE IV. COST ESTIMATION

System Component	Cost (€)
Heater	6 089.5
Regenerator	4 916.5
Cooler	2 509.8
Engine bulk	7 884.4
Total capital cost of Stirling engine	21 400.0

According to the results, the total cost of the equipment is 21 400€. This value is relatively close to the capital investment cost of a Solo Stirling 161 (25 000€). Considering the cost of each system component, the heater, the regenerator, the cooler and the engine represent, respectively, 28.5%, 23.0%, 11.7% and 36.8% of the total cost. Thus, the heater and the engine bulk are the most costly components.

The cost equations presented in this work allow the combined variation of size and performance aspects. Therefore, varying the operational and the geometric characteristics of the Stirling engine and optimizing the costs of the system, seems to be the best commitment in optimizing these thermal plants. For instance, there is an optimal value for the internal diameter of the heater pipes for which the engine efficiency is maximum (see Fig. 10).

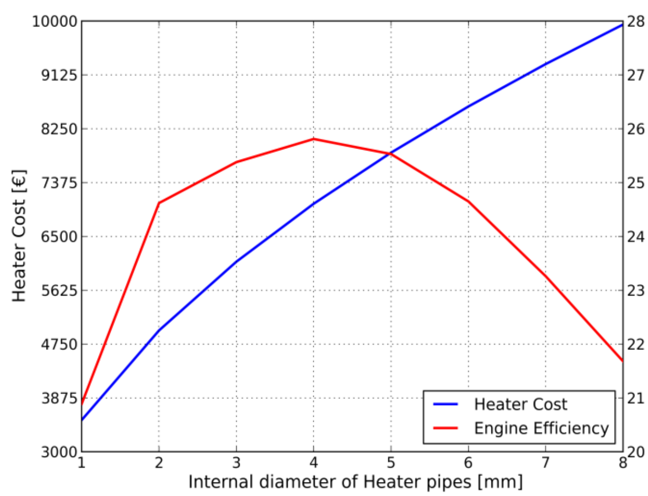

Fig. 10 Stirling engine efficiency and heater cost as a function of the internal diameter of the heater pipes.

According to the results, the Stirling engine efficiency is maximum (i.e. 25.8%) when the internal diameter of the pipes is 4 mm. For this geometrical input, the heater cost corresponds to 7 031.6 €.

V. Conclusion

In this work, the mathematical modelling of a Stirling engine is presented in order to study its performance. For this purpose, numerical simulations for non-ideal engine working conditions were performed, including the heat transfer limitations and pumping losses throughout the Stirling thermodynamic cycle. The paper also discloses a methodology to estimate the costs of the system. The system total cost was decomposed in four cost equations, representative of the heater, cooler, regenerator and engine bulk, respectively.

Results show that heat-transfer limitations strongly affect engine efficiency, particularly in the regenerator case. The pumping losses increase with gas mean pressure and engine rotational speed. The heat exchangers effectiveness is slightly higher for helium when compared with hydrogen.

Considering the defined input parameters, the total capital costs are close to real commercial models for similar applications. Plus, the engine bulk and the heater are the most expensive components.

Stirling engines have been identified as a promising technology for the conversion of primary energy into useful power due to their high efficiency levels, low pollutant emissions, low noise levels and mostly due to their flexibility in terms of fuel sources. The possible use of a renewable energy source is very important from the point of primary energy savings and reduction of carbon emissions.

The main purpose behind this study is the definition of a thermal-economic model applied to a cogeneration system for a residential application. The system to be modeled is based on Stirling engine technology combined with a solar collector as a renewable energy source. After defining the cost equations for the system components, they should be integrated in the thermal-economic optimization model in order to achieve the best technical and economic output of the system under analysis.

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