An integrated risk management model for an oil and gas company

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Abstract—Oil and gas companies’ returns are heavily affected by price fluctuations. In financial terms, the “price in—price out” dynamics influence companies’ gross margins and impact to a high extent on their multイヤyear budgets and accomplishment of goals. Due to world scale size and geographically scattered organizations oil and gas companies use separate hedging tactics to protect each of their business units (e.g. crude oil production, oil refining and natural gas) from the risk associated with the fluctuation of prices. The present research compares, for an oil and gas company, the results of using a “hedging at business unit level” approach with the results of employing a “hedging at company level” approach, by finding the best derivatives portfolios through coherent risk measures and stochastic optimization. The analysis is subsequently extended to a utility based approach, where the company’s risk tolerance is included.

Keywords—copula’s functions; Monte Carlo simulation; risk measures; portfolio optimization

I. INTRODUCTION

Oil and gas (O&G) companies’ returns and strategies are substantially affected by the price fluctuations of crude oil, natural gas and refined products (e.g. gasoline, diesel, fuel oil and aromatics), which induce these companies to find ways to minimize price risk exposure and the inherent gross margin uncertainty. A risk management methodology provides a significant degree of protection from extreme price movements, but it should not be expected to remove all the risk to which an oil and gas company is exposed. In particular, it can create liquidity risks and counterparty performance risks [1]. All O&G companies use derivatives instruments, like swaps and options, to share price risks with other counterparties, usually through the world largest financial groups, specialized in derivatives arbitrage. The amount of money involved in these derivatives at world level reaches several times the value of the underlying physical assets [2], with more than 3 trillions of dollars on daily open contracts. Black and Scholes [3] research on options pricing and the late 1970’s deregulation of the United States energy markets provided the ingredients for the steady growth of derivatives in the energy markets, as stated by a landmark report [4].

Markowitz [5] original work on diversification of investments and selection of efficient portfolios, later known as the “modern portfolio theory”, potentiated the derivatives use along the physical energy trade, reducing the companies price risk exposure. Artzner, Delbaen, Eber and Heath [6] and Rockafellar and Uryasev [7] stated the foundations for the use of Conditional Value-at-Risk (CVaR) as a coherent risk measure for portfolio risk optimization, going further the standard deviation measure used by [5] and the Value-at-Risk (VaR) proposed by [8] RiskMetrics methodology. Von Neumann and Morgenstern [9] defined four axioms for the utility as a decision criterion, assuring solid ground for long term research and [10] showed, for various utility functions and empirical returns distributions, that the expected utility maximizer could typically do very well if he acted knowing only the mean and variance of each distribution. More recently [11], found significant differences in the optimal energy hedge strategies based on the utility function chosen. [12] presented the first risk analysis in capital investment using Monte Carlo simulation. [13] made an extensive study on risk, uncertainty and investment decision-making in the upstream oil and gas, concluding that the companies using the most recent approaches performed better than their competitors which used “old” techniques.

This paper considers the physical commodities and potential derivatives portfolio for a European oil and gas medium sized company, with equilibrated assets of crude oil productions, oil refining and natural gas. We start by minimizing risk exposure considering one portfolio for each of the three business units (oil exploration, oil refining, and natural gas). In a second phase, we extend the method considering only one integrated portfolio for all the three business units. In the last phase, we optimize the integrated portfolio with a utility based optimization, where the company’s risk tolerance is included. The final goal is to evaluate how these three approaches perform in order to manage company price risk exposure.

The remainder of this paper is organized as follows. Section II describes the price variables stochastic modeling for gross margin simulation, built on Copula’s functions to model the correlation between the prices of the products. Section III presents a brief overview on the most common risk measures used to evaluate companies’ exposure to losses. Section IV proceeds with the formulation of the portfolio, combining physical trade and derivatives payout in order to minimize the risk exposure. Section V describes three approaches for portfolio risk optimization and finds the hedging portfolios that minimize gross margin variability: (1) through a defined risk measure at business unit level, (2) through a defined risk measure at company level (3) the risk measure being...
incorporated in a utility function reflecting the company-wide risk limits and comparing it with the previous defined risk measure. Section VI presents some final remarks and proposals for future research.

II. STOCHASTIC MODELING OF PRICES

Historic yearly price variability of crude oil, natural gas and refined products is significant, as observed in Fig. 1. This variability is captured by measuring variability in terms of “returns” rather than absolute price movements, where returns are calculated as the natural logarithm of the ratio of current month average price over last month average price. The distributions of returns for many commodities is in fact normally distributed, which means that the corresponding price variable is lognormal distributed, as stated by [1]. Our proposed price risk exposure methodology is applied to a long term period (more than three years) according the company’s defined strategy, meaning the derivatives prices to be agreed with the counterparty will have a yearly fixed price for each physical asset (crude oil, natural gas, refined products and refining margin). The commodities price uncertainty is modeled through the unconditional Equally Weighted Moving Average Methodology of the historical spot price returns data as tested by [14]. We use 12-months moving average spot price returns, from 1990 till 2012, with price data from Platt’s, e.g. for Brent (the crude oil reference in Europe), gasoline premium unleaded (gasoline 10 ppm), ultra low sulphur diesel (diesel 10 ppm). For futures and options pricing we use Reuter’s data, from 2012 till 2016.

As one can see in Fig. 1, a parametric function (e.g. a normal distribution or a student’s $t$ distribution) is not suitable to properly model the prices behavior in the tails, underestimating the rare events (known as “fat tails”).

![Figure 1. Probability density function for 12-months moving average spot price returns (%) for Brent and two refined products (1990-2012). Source: http://www.platts.com.](image)

Rosenblatt [15] introduced non-parametric data fitting with kernel functions, [16] apply kernel distributions to integrate market, credit, and operational risks and to model the total economic capital required to protect a financial institution against possible losses. In our study we follow the kernel approach, using a Gaussian kernel to fit the prices of each product (e.g. Brent, natural gas, diesel, gasoline, propane, fuel- oil, etc.), as described by [17]. The kernel fitting result for Brent historical prices is shown in Fig. 2.

![Figure 2. Gaussian kernel fitting for 12-months moving average spot price returns (%) for Brent.](image)

Modeling correlation between the prices of the different products, assuring nonlinear dependencies are satisfied, leads us to copula’s functions. [18] theorem provides the theoretical foundation for the application of copulas’ functions. The great advantage of copula’s functions is to allow the correlation pattern (the copula function) to be independent from the random variable $X$, marginal’s. We can have different marginal distributions for each $X$ (normal, student’s $t$, or other) and join them with a correlation structure modeled by one of the copula functions (Clayton, Frank, Gumbel, Normal, student’s $t$ are the most used). This is a significant step ahead on the classical covariance matrix where no independence exists between marginal’s and the correlation structure [19]. Historical simulation (assuming that there is no relation between variables) or an analytical approach is not a viable solution to this case, due to the complexity of the business system. [20] presents an example applied to O&G exploration, showing how copula functions along with Monte Carlo simulation help to fill the gap of poorly defined correlations between events, with heavy impacts on the projects risk exposure estimation. Monte Carlo simulation is the most suited method for portfolio analysis [21] and [22], whereas the copula’s functions are the most flexible tool in order to model a multiple risks portfolio [23]. For this research the gauss copula (an elliptical copula family) derived from the multivariate normal distribution proved the best fit solution using the “Schwarz information criterion” (known as “Bayesian information criterion”).

III. RISK MEASURES

Holton [24] review financial risk concepts from past authors as [25], [26], [5] and states that risk entails two components: uncertainty and exposure. Uncertainty can be represented through the appropriate use of probability [27]. Probability refers to the likelihood of facing a particular event. The frequentist (or objective) interpretation of probability is based on the long-run relative frequency of an event. The subjective interpretation of probability is based on an individual’s degree of belief that a particular event will occur, [27]. Exposure (also called impact ([28])) is the foreseen potential loss in money or in other measurable variable if the risk occurs. Note that an impact can also be positive, whereas exposure is associated with the notion of a negative impact. The importance of confronting an O&G gross margin “exposure” with a measure of the respective “uncertainty” is to guarantee that a company meets its obligations with a previous imposed degree of confidence. If a company’s debt obligations go under a given limit with a probability of $p\%$, ....
this means that the company has at least $p\%$ of probability of not meeting its obligations (i.e., it is an overexposed company, [29]. To overcome this threat, an overexposed company can hedge some of its risks, change its business portfolio or capital structure, being the latter two, long-term choices. On the opposite side, if all company’s obligations are much higher than its downside risk measure, this means that the company is not sharing its risks i.e., it is an over-insured company. An over-insured company can overcome this situation by increasing its debt (sharing the risk with counterparties) or by repurchasing shares. [30] and [31] address the benefits of a long-term hedging on commodity prices, leading to a reduction of the risk premium that the company must pay for its debt capital, and referring that hedging contributes to increased confidence by the investors as regards debt redemption. [32], confronts company deterministic obligations with their gross margin probabilistic nature, as presented in Fig. 3.

A solid company risk management implies that company’s business portfolio simulation should be done in an integrated way, with all the business units facing the same market risks, through a simultaneous impact, as referred by [33].

The most elementary risk measures are symmetric mean dispersion measures: standard deviation, absolute deviation and coefficient of variation, which all have the limitation of measuring in the same way the down and up side of the risks. This is particularly erroneous if the gross margin distribution is not symmetric. Companies’ concerns are more related with a downside measure of risk, implying that semi-variance or semi-deviation are more suited for this purpose. However these measures reflect the variability of the mean of the variable, not quantifying the probability of falling below a given critical value. “RiskMetrics” methodology was created by J.P.Morgan [8] with the purpose of having a measure of the bank portfolio exposure to market volatility and introduced the “Value-at-Risk” (VaR) measure. VaR is a downside risk measure defined as “the worst expected loss at a $q\%$ confidence level over a given $h$ period, under normal market conditions. The most common values for $q\%$ are 95% or 99%. However VaR reflects the worst expected loss, telling nothing about “how bad” are the events below the limit value at $q\%$ confidence level.

Artzner, Delbaen, Eber and Heath [6] defined the axioms necessary and sufficient for a measure of risk to be coherent: positive homogeneity, translation-invariance, monotonicity and sub-additivity. Rockafellar and Uryasev [35] proof standard deviation and VaR are not coherent measures since the first violates translation-invariance and monotonicity, while VaR fails sub-additivity. They propose “Conditional Value-at-Risk” (CVaR) defined in (1) as a coherent risk measure, also known latter as “Expected Tail Loss” (ETL) or “Expected Shortfall”, assuring the cornerstone sub-additivity property, $Risk(A+B) \leq Risk A + Risk B$ and permitting a clear measure of how large are losses deep into the left tail, as presented in Fig. 4:

$$CVaR_{[1-\alpha]} = E\left( X_{\alpha} \leq \text{VaR}_{[1-\alpha]} \right)$$  \hspace{1cm} (1)

where $X_{\alpha}$ is the value defined for having VaR for a level of confidence of $(1-\alpha)$.

![Figure 3. Company’s gross margin exposure to main obligations (Capex: capital expenditure, Opex: Operating Costs)](image)

![Figure 4. Risk measures: Value-at-Risk and Conditional Value-at-Risk](image)

IV. PROBLEM FORMULATION

Derivatives are financial instruments (contracts) that do not represent ownership rights of any asset but rather derive their value from the value of the underlying commodity.

A European option which conveys the right to buy something at a specific price is called a “call”; an option which conveys the right to sell something at a specific price is called a “put”. Detailing: call/put is a contract where the call/put buyer pays a front price (premium) with the right to buy/sell an underlying asset if it rises/falls above/below the pre-agreed “strike” price, at an agreed date. The call/put seller receives the “premium” upfront price and has the obligation to sell/buy an underlying asset if it rises/falls above/below the pre-agreed “strike” price at an agreed date. For example, one call option to buy a thousand cubic feet of natural gas at a price of $4.60 in December 2002 (strike price) may cost $0.73. If the price in December exceeds $4.60, the call buyer (usually a company gas producer) can exercise his option and buy the gas for $4.60. More commonly, the call seller (usually a company gas producer) pays the call buyer the difference between the market price and the strike price. If the natural gas price is less than $4.60, the buyer lets the option expire and loses $0.73 which in fact is the premium to avoid undesirable scenarios.

As example, one put option to buy X thousands crude oil barrels at a unitary price of 80 $/barrel (strike price) in December 2002 may cost 10 $/barrel. If the price in December is below 80 $/barrel the put buyer (usually a crude oil producer) can exercise his option and sell X thousands crude oil barrels at 80 $/barrel, no less. The put seller (usually a refinery) pays the put buyer the difference between the market price and the strike price. If the crude oil price is more than 80
where $S_i$ and $B_i$ are respectively the selling and buying price indexes, $Y_i$ and $W_i$ are the selling and buying formula weights and $Q_{NG}$ is the total quantity in kWh.

For the R&D business unit (5):

$$GM_{R&D} = \sum_{i=1}^{4} Y_i P_i - P_{BR} Q_{R&D}$$

(5)

where $Y_i$ are the yields of each $i$ refined product, $P_i$ are the unitary refined products prices, $P_{BR}$ is the is the unitary Brent price and $Q_{R&D}$ is the quantity refined (in tonnes).

The result of the derivative payout (6):

$$Der_{Payout} = \sum_{i=1}^{4} \left[ \alpha (S_i - X_i) + \beta (S_i - X_i | S_i > X_i - Pr_C) + \gamma (X_i - S_i | S_i > X_i) + Pr_C \right] Q_i$$

(6)

Where $\alpha$, $\beta$, $\gamma$ ($\%$) are the notional amount of swaps, puts and calls to be hedged as a percentage of the each $i$ item physical quantity, $Q_i$ is the quantity (in tonnes) of each item to be hedged, $X_i$ is the price of item $k$ at maturity (item $k = Brent$, natural gas, refined products or the refining margin, in the case of R&D) $S_i$ is the swap price for item $k$, $S_i^p$ is the put strike price for item $k$, $S_i^c$ is the call strike price for item $k$, $Pr_C$ is the put premium for item $k$, $Pr_C$ is the call premium for item $k$.

V. RISK OPTIMIZATION

As proposed by [7], and more recently by [34], a more robust portfolio optimization than the variance minimization is to minimize the Conditional Value-at-Risk (CVaR). Adapting the [7] method, we will minimize the CVaR of the company’s gross margin, assuming a confidence level of 95% and subject to a payout limit “$L_{Company}$” reflecting the maximum payout the company can afford. In fact, a given company can hedge only up to an amount that the counterparties (i.e., the banks) consider trustworthy, after checking the company’s financial ratios and international rating (by rating agencies such as Standard & Poors, Fitch or Moodys).

In program 1 we start with the optimization made separately for each business unit (BU), and go through minimizing CVaR using the gross margin symmetric as a “loss” distribution. The payout limit $L$ defined for each BU, $L_{BU}$ is proportional to the respective gross margin and all the BU payout sum $\sum L_{BU}$ is not greater than the maximum payout defined for the entire company, $L_{Company}$. The formulation to be solved for each BU is:

Program 1:

$$\text{Minimize } CVaR_{sym}(GM_{BU} + Der_{PayoutBU} f(\alpha, \beta, \gamma))$$

(7)

Subject to:

$$Der_{Payout} \leq L_{BU}$$

(8)
The formulation for the optimal integrated hedge, done simultaneously with all businesses units is:

Program 2:

\[
\text{Minimize } \text{CvaR}_\text{BU} \left( \sum_{i=1}^{3} \left( G_{\text{BU}_i} - G_{\text{BU}_i} + \text{Der}_{\text{Payoff}} f(\alpha, \beta, \gamma) \right) \right)
\]

Subject to:

\[
\text{Der}_{\text{Payoff}} \leq L_{\text{Company}},
\]

\[
\alpha, \beta, \gamma \leq 1 \text{ (to avoid over hedging).}
\]

Applying Rockafellar and Uryasev [35] properties for CVaR, a quadratic programming solves the CVaR minimization problem with stochastic programming (see [36], [37] and [38]). We used ModelRisk [39] software for simulation and Optquest [40] for stochastic optimization, as presented by [41].

Table 1 shows the results for Program 1 and Program 2. The “Total” column was achieved, for program 1, with the joint simulated result from each business unit optimal individual hedge.

Table 1 Hedging at business unit level versus hedging at company level

<table>
<thead>
<tr>
<th>Gross Margin MUS</th>
<th>R&amp;D</th>
<th>NG</th>
<th>E&amp;P</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Hedge</td>
<td>439</td>
<td>76</td>
<td>101</td>
<td>516</td>
</tr>
<tr>
<td>CVaR (95%)</td>
<td>-469</td>
<td>-36</td>
<td>151</td>
<td>354</td>
</tr>
<tr>
<td>Business Unit level</td>
<td>247</td>
<td>26</td>
<td>20</td>
<td>253</td>
</tr>
<tr>
<td>CVaR 95% hedge Program 1</td>
<td>-95</td>
<td>3</td>
<td>305</td>
<td>330</td>
</tr>
<tr>
<td>CVaR (95%)</td>
<td>177</td>
<td>58</td>
<td>61</td>
<td>298</td>
</tr>
<tr>
<td>Company level</td>
<td>14</td>
<td>1</td>
<td>234</td>
<td>443</td>
</tr>
<tr>
<td>CVaR 95% hedge Program 2</td>
<td>32%</td>
<td>31%</td>
<td>35%</td>
<td>28%</td>
</tr>
</tbody>
</table>

Table 1 shows that for program 1, adding up the CVaR for each BU, is overestimating the downside risk by $115 \times 10^6$ when compared with the result of simulating the whole company with each BU optimal solution (15):

\[
\sum_{\text{BU}=1}^{3} \text{CvaR}_\text{BU} = -95 + 5 + 305 = 215 < \text{CVaR}_\text{BU} = 330
\]

However, there is a wider and clarifying CVaR difference for the “Company level hedge” program 2 solution (16):

\[
\sum_{\text{BU}=1}^{3} \text{CvaR}_\text{BU} = 14 + 1 + 234 = 249 < \text{CVaR}_\text{BU} = 443
\]

The CVaR solution obtained with “business unit level hedge” reveals that in the worst 5% scenario the company can make at least $330 \times 10^6$, while the CVaR obtained with “Company level hedge” reveals that in the worst 5% scenario the company can make at least $443 \times 10^6$. So, this means that the solution achieved at “business unit level hedge” is overestimating the risk by $113 \times 10^6$ (i.e. 34% more).

These results are in accordance with Rosenberg and Schuermann [42] for an integrated risk management approach for one internationally active bank, aggregating different risk types, using the method of copulas, capturing marginal distributions details, such as skewness and fat-tails, concluding that conventional methods through a non-portfolio approach has no diversification benefits and overestimates risk by more than 40%.

In our situation hedging at company level reveals the beneficial diversification effects that are hidden at a business unit level hedge.

If we look at standard deviation instead of CVaR, the risk overestimated will be $63 \times 10^6$ (i.e. 23% more), which confirms [7] and [35], who referred to CVaR as a more trustworthy risk measure. Fig.5 shows the gross margin distribution for the three situations presented in Table 1.

We will now take the previous best approach, “Company level”, with a new objective function, which, instead of CVaR, will maximize the company’s utility, through the exponential utility function. Exponential utility function is a well-tested way to guide O&G decisions, as referred by Walls [43] in selecting investments in oilfields according to company risk tolerance. Delquie [44] shows an interpretation of risk tolerance as the maximum loss the decision maker is willing to be exposed to at a stated probability level, regardless of the upside potential and Howard [45] estimates the corporate risk tolerance being about one sixth of equity.

As referred by Pratt [46] the exponential utility function defines a risk-averse decision maker, not depending on his initial wealth, being defined as (17):

\[
\text{Utility} = \frac{e^{\theta \text{CvaR}} - 1}{\theta}
\]
where $\rho > 0$ is the company’s risk tolerance, defined as the amount that the decision maker accepts to play a game with a 50% probability of winning $\rho$ and a 50% probability of losing half of this amount, $\rho/2$.

Risk tolerance is $\rho = 1/\beta$, where $\beta$ is the Pratt [46] measure of absolute risk aversion.

Half the risk tolerance, by definition, is the amount that the company can accept losing with a 50% chance. For consistence the maximum payout amount assumed in this optimization is equal to half the risk tolerance,

$$\frac{\rho}{2} = L_{\text{Company}}$$

Maximizing an utility function is equivalent to maximizing the respective Certainty Equivalent (CE) as referred by [47], [48] and the CE for the exponential utility function, [27], is the expected value $\mu$, discounted by a fraction (risk discount) proportional to the variability of the returns and inversely proportional to the agent risk tolerance, $\rho$.

$$CE = \mu(x) - \left(\frac{\sigma^2}{2\rho}\right)$$

applying for the company gross margin (GM):

$$CE = \mu_{GM_{\text{Company}}} - \left(\frac{\sigma^2_{GM_{\text{Company}}}}{2\rho_{\text{Company}}}\right)$$

Recent work from Street [49] and Andrieu, Lara and Seck [50], for relevant electricity producers, incorporates risk measures constraints (such as CVaR) into a classic “expected return maximization problem”, concluding this approach can be considered equivalent to a “generalized expected utility agent maximization problem”. In our next phase, along with the restriction of the company maximum payout, we will exchange the CVaR minimization by the Certainty Equivalent maximization of the portfolio constituted by the physical and derivative assets.

Program 3:

Maximize $CE\left(GM_{E&P} + GM_{NG} + GM_{R&D} + Der_{\text{Payout},f}(\alpha, \beta, \gamma)\right)$

Subject to:

$$Der_{\text{Payout}} \leq L_{\text{Company}},$$

$$\alpha, \beta, \gamma \leq 1$$ (to avoid over hedging).

Table 2 presents the results from program 2 (minimizing CVaR at the company level) and for program 3 (maximizing CE at the company level).

Table 2. Company level hedging: minimizing CVaR versus maximizing CE

<table>
<thead>
<tr>
<th>Program</th>
<th>Min CVaR@95%</th>
<th>CE</th>
<th>R&amp;D</th>
<th>NG</th>
<th>E&amp;P</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Program 2</td>
<td>14</td>
<td>1</td>
<td>234</td>
<td>443</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Program 3</td>
<td>54</td>
<td>2</td>
<td>259</td>
<td>470</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Regarding standard deviation, no significant difference exists between the two optimization methods ($208\times10^6$ versus $209\times10^6$), which confirms again CVaR as a more trustworthy risk measure than standard deviation.

Regarding CVaR, the “Company level hedge” CVaR minimization evaluates CVaR$_{95\%}$ at $443\times10^6$, while “Company level hedge” CE maximization evaluates CVaR$_{95\%}$ at $470\times10^6$, meaning that the former method is overestimating risk by $27\times10^6$ (i.e. 6% more). Taking into account company risk profile through the Certainty Equivalent maximization lead us to slightly less overestimated risk than taking minimizing CVaR.

In Fig. 6 we show the close shape between both referred methods: minimizing company level CVaR and maximizing company level “Certainty Equivalent”, supporting Andrieu, Lara and Seck [50] on the analytic similarities found in both methods, but we state differences in the left lower tail.
VI. FINAL REMARKS

The present research concludes that making an optimization hedging at company level is more effective than doing it at each business level, as is still done nowadays by the great majority of the O&G companies. As referred by Vasey [51] many of the energy trading, transaction, and risk management (ETRM) software were originally developed for the financial industry and still do not reflect the complexity of the energy business, operating under multiple regulatory environment where physical and derivatives products under the same portfolio demands highly customized solutions. However, in the last couple of years, some software providers have launched new integrated risk trading desks, so we believe it is a matter of time until risk integration would become a current practice. This paper evaluates the gains to be achieved with an integrated price risk management for an Oil and Gas company with a common portfolio of physical and derivatives assets, optimized through coherent risk measures and incorporating the company risk tolerance.

The use of utility based approach to business portfolio optimization is not common in the O&G companies [13]. The assumptions in this paper assume that the risk tolerance is, for simplicity at this stage, the double of the maximum payout. Future research, given this promising result, should relax this elementary assumption and test other utility functions, preferably custom made for the real company risk profile.

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